Econ. 557
Yale University

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Time Series Econometrics II: Unit Roots and Cointegration

Take Home Examination

Answer ALL Questions: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 29 May 1998, 12:00 noon.

Question A.

The time series X_t $(t = 1, ..., n)$ satisfies

$$\Delta X_t = u_t = C(L)\,\varepsilon_t, \quad C(L) = \sum_{j=0}^{\infty} c_j L^j, \quad \sum_{j=0}^{\infty} j^{\frac{1}{2}} |c_j| < \infty \tag{1}$$

where $\varepsilon_t \equiv iid(0, \sigma^2)$ and where $X_0 = O_p(1)$ as $n \to \infty$

Suppose the following linear trend regression model is fitted

$$X_t = \widehat{b}t + \widehat{u}_t. \tag{2}$$

A robust (HAC) standard error, s_b, and robust t-ratio, $t_b = \frac{\widehat{b}}{s_b}$, for $\widehat{b}$ are computed to allow for serial correlation in the residuals $\widehat{u}_t$ by using conventional lag kernel techniques.

1. Find the asymptotic behaviour of the t-ratio t_b as $n \to \infty$.
2. Discuss the implications of your results.
3. Perform a simulation experiment to illustrate your findings.

Question B.

For the time series X_t in Question A, consider the following unit root estimator

$$\widehat{a} = \frac{\sum_{t=1}^{n} \operatorname{sign}(X_{t-1})\, X_t}{\sum_{t=1}^{n} |X_{t-1}|}$$

where $\operatorname{sign}(X_{t-1}) = 1, -1$ according as $X_{t-1} \geq 0$, $X_{t-1} < 0$. Suppose $C(L) = 1$, so that $u_t = \varepsilon_t$.

1. Show that $\widehat{a}$ is an instrumental variable estimator.
2. Find the limit behaviour and limit distribution of $\widehat{a}$ as $n \to \infty$.
3. Construct a t-ratio statistic for $\widehat{a}$ to test for a unit root and find its limit distribution.
4. Discuss the implications of your findings and see if you can extend them to an ADF type regression of finite order p to cover the finite autoregressive nuisance parameter case where $C(L) = 1/b(L) \neq 1$.
5. Perform a simulation experiment to compare your new unit root test with tests based on the usual least squares estimator.

Question C.

1. Download the Nelson Plosser (1982) macroeconomic data set and extended Nelson Plosser data (Schotman and van Dijk, 1991) from my web site at URL: http://korora.econ.yale.edu . The data are annual historical time series for the US over the period 1860-1980. You may use either data set and confine your attention to a few of the major series.

2. Estimate the fractional integration parameter d for the series by log periodogram regression using a components model of the form

$$y_t = a + bt + y_t^0, \quad t = 1, ..., n \tag{3}$$
$$(1 - L)^d y_t^0 = u_t \tag{4}$$

3. Assuming that the usual least squares regression formula delivers the correct asymptotic standard error of the log periodogram regression estimate of d, construct 95% confidence intervals for d for each of the series you consider.

4. Discuss the implications of your results for the presence or absence of a unit root stochastic trend in each of the series.

5. Comment briefly on the efficient estimation of b in (3).

References

Nelson, C. R. and C. Plosser (1982). "Trends and random walks in macroeconomic time series: Some evidence and implications," *Journal of Monetary Economics* 10, 139–162.

Schotman, P. and H. K. van Dijk (1991). "On Bayesian routes to unit roots," *Journal of Applied Econometrics*, 6, 387-402