Econ. 557b
Yale University

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Time Series Econometrics II
Unit Roots and Cointegration

Take Home Examination

Time Allowed: Three weeks
Instructions: Answer at least one Question.
Due Date & Time: Monday 10 May 2004, 12:00 noon.

Question 1.

In the model

$$y_t = \alpha y_{t-1} + u_t, \;\; u_t = g\left(\frac{t}{n}\right)\varepsilon_t, \quad \alpha = 1, \tag{1}$$

the time series $\{y_t : t = 0, 1, ...n\}$ is observed, $y_0 = O_p(1)$, $g(\cdot)$ is a (non-random) continuously differentiable function on the interval $[0, 1]$, and the error ε_t is stationary with Wold representation $\varepsilon_t = \sum_{j=0}^{\infty} c_j e_{t-j}$, where $e_t = iid\,(0, \sigma^2)$ with finite fourth moments and where the coefficients c_j satisfy $\sum_{j=0}^{\infty} j|c_j| < \infty$.

(a) Find the limit behavior of $y_{[nr]}/\sqrt{n}$.

(b) It is proposed to test y_t for a unit root using the following test statistics

$$\begin{aligned} Z_\alpha &= n(\hat{\alpha} - 1) - \frac{\hat{\lambda}}{n^{-2}\sum_{t=1}^{n} y_{t-1}^2}, \\ L_n &= \frac{y_n^2}{n\hat{\sigma}_u^2}, \end{aligned}$$

where $\hat{\alpha} = \sum_{t=1}^{n} y_t y_{t-1} / \sum_{t=1}^{n} y_{t-1}^2$, and the residuals $\hat{u}_t = y_t - \hat{\alpha} y_{t-1}$ are used to construct

$$\hat{\sigma}_u^2 = \frac{1}{n}\sum_{t=1}^{n} \hat{u}_t^2,$$

and the following kernel estimate

$$\hat{\lambda} = \sum_{h=1}^{M} k\left(\frac{h}{M}\right) C_{\hat{u}}(h), \quad C_{\hat{u}}(h) = \frac{1}{n}\sum_{1 \le t, t+h \le n} \hat{u}_t \hat{u}_{t+h}, \tag{2}$$

for some kernel function $k(x)$ that is continuously differentiable to the second order and satisfies $k(-x) = k(x)$, $k(0) = 1$, $|k(x)| \le 1$ and $\int_{-1}^{1} k(x)^2 dx < \infty$. The parameter M in (2) satisifies $\frac{1}{M} + \frac{M}{n} \to 0$ as $n \to \infty$. Find the limit distribution of the two statistics Z_α and L_n as $n \to \infty$.

(c) Can Z_α and L_n be used to test the hypothesis $\mathcal{H}_0 : \alpha = 1$ in (1)? Describe how the tests would be conducted and any possible problems you see in the implementation and use of the tests.

(d) Conduct a simulation experiment to study the finite sample size (using nominal critical values based on the usual asymptotic theory for these tests where g is a constant function) and power of the two tests for some selected non-constant valued functions $g(\cdot)$.

(f) Briefly discuss any other ideas you may have concerning how to conduct a unit root test in a model like (1).

(g) Can you think of an empirical context where a model like (1) might be appropriate?

Question 2.

Take at least one of the exercises given in lectures and work out a full solution to the asymptotic theory. If this is relevant to one of the exercises you consider, you may also conduct a brief simulation study to illustrate how well the limit theory in your chosen exercise works in finite samples.

Question 3.

Do any empirical study of unit roots and cointegration using an economic model, data and methods of your choice. Write up your results as a formal report, including graphics and empirical findings.