Econ. 557b | Peter C. B. Phillips
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Time Series Econometrics II
Unit Roots and Cointegration

Take Home Examination

Time Allowed: Six weeks
Instructions: Answer at least one Question.
Due Date & Time: Friday 10 May 2002, 12:00 noon.

Question 1. (Theory: Testing in Cointegrated Models)

(a) In the cointegrated system

$$y_t = \alpha + \beta' x_t + u_{0t}, \tag{1}$$
$$\Delta x_t = u_{xt}, \tag{2}$$

the error vector $u_t = (u_{0t}, u_{xt}')'$ is a jointly stationary time series with Wold representation $u_t = \sum_{j=0}^{\infty} C_j \varepsilon_{t-j}$ where $\varepsilon_t = \text{iid } (0, \Sigma)$ with finite fourth moments and the coefficients C_j satisfy $\sum_{j=0}^{\infty} j^{\frac{1}{2}} ||C_j|| < \infty$. Suppose the long run variance matrix of u_t is the block diagonal matrix

$$\Omega = \begin{bmatrix} \omega^2 & 0 \\ 0 & \Omega_{xx} \end{bmatrix},$$

where the partition is conformable with that of u_t.

(b) It is proposed to estimate (1) by least squares regression using observations over $t = 1, ..., n$. The residuals from this regression, $\hat{u}_{0t} = y_t - \hat{\alpha} - \hat{\beta}' x_t$, are used to construct the following estimate of ω^2

$$\hat{\omega}^2 = \sum_{h=-n+1}^{n-1} k\left(\frac{h}{n}\right) C_{\hat{u}_0}(h), \quad C_{\hat{u}_0}(h) = \frac{1}{n} \sum_{1 \leq t, t+h \leq n} \hat{u}_{0t} \hat{u}_{0t+h}, \tag{3}$$

where $k(x)$ is a kernel function that is continuously differentiable to the second order, satisfying $k(-x) = k(x)$, $k(0) = 1$, $|k(x)| \leq 1$ and $\int_{-1}^{1} k(x)^2 dx < \infty$. [Note that the definition of $\hat{\omega}^2$ given in (3) does not involve a bandwidth parameter].

(c) Find the limit of $\hat{\omega}^2$ as $n \to \infty$ and briefly comment on your finding.

(d) It is proposed to test the hypothesis $H_0 : R\beta = r$ against $H_1 : R\beta \neq r$ by using the Wald statistic

$$W_n = \left(R\hat{\beta} - r\right)' \left[R M_{xx}^{-1} R'\right]^{-1} \left(R\hat{\beta} - r\right) / \hat{\omega}^2, \tag{4}$$

where

$$M_{xx} = \sum_{t=1}^{n} (x_t - \bar{x})(x_t - \bar{x})'.$$

Find the limit distribution of this statistic as $n \to \infty$. Compare your finding with the limit distribution of the usual cointegrated regression Wald test in which ω^2 is estimated by a consistent kernel estimate.

(e) Find the limit distribution of the statistic W_n under the local alternative hypothesis $H_0 : R\beta = r + \frac{c}{n}$ for some $c \neq 0$.

(f) Indicate how this approach to statistical testing in cointegrated regressions generalizes when Ω is not block diagonal and (1) is estimated by FM-OLS regression.

Question 2.

(a) Simulation

Conduct a simulation study to obtain critical values and examine the power of the test statistic W_n in Parts 1(a) and 1(f).

(b) Empirical Study

On data and a model of your choice, apply the test W_n and its FM generalization to assess evidence in support of a hypothesis about β. [For example, you could test whether the coefficient $\beta = 1$ in a cointegrating regression of aggregate consumption on income].

Question 3.

Do any empirical study of unit roots and cointegration using an economic model, data and methods of your choice. Write up your results as a formal report, including graphics and empirical findings.