

Econ. 557b  
Yale University

Peter C. B. Phillips  
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# Time Series Econometrics II Unit Roots and Cointegration

## Take Home Examination

*Time Allowed:* Six weeks

*Due Date & Time:* Friday 25 May 2001, 12:00 noon.

### Part 1. (Testing a Null of Cointegration)

- (a) Develop a KPSS-type test of the null hypothesis of cointegration that the time series  $y_t$  and  $x_t$  are related by a model of the form

$$y_t = \beta' x_t + u_{0t}, \quad (1)$$

$$\Delta x_t = u_{xt}, \quad (2)$$

where  $u_t = (u_{0t}, u'_{xt})'$  is a jointly stationary time series with Wold representation  $u_t = \sum_{j=0}^{\infty} C_j \varepsilon_{t-j}$  where  $\varepsilon_t = \text{iid } (0, \Sigma)$  with finite fourth moments and the coefficients  $C_j$  satisfy  $\sum_{j=0}^{\infty} j^{\frac{1}{2}} \|C_j\| < \infty$ .

- (b) Develop an asymptotic theory for your test statistic, show that it is invariant to nuisance parameters and investigate its asymptotic properties under the null hypothesis of cointegration and the alternative of no cointegration.
- (c) Briefly indicate what happens to your test statistic when deterministic regressors (such as an intercept or a trend) appear in (1).

### Part 2. (Simulation)

Conduct a simulation study to obtain critical values for the test statistic you obtained in Part (a).

### Part 3. (Empirical Study)

On data of your choice:

- (a) Apply your test to assess evidence in support of cointegration;
- (b) See if the results of your test are compatible with those of a residual based test of a null of no cointegration.