Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer ALL Questions: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 14 January 2000, 12:00 noon.

Question A.

Part 1

Start by assuming that the time series X_t is generated by the model

$$X_t = \alpha + \beta \log t + u_t, \quad t = 1, ..., n \tag{1}$$

where α and β are unknown parameters whose least squares regression estimates are denoted by $\widehat{\alpha}$ and $\widehat{\beta}$, respectively. The error u_t in (1) is assumed to be *iid* $(0, \sigma^2)$ with finite fourth moment.

1. Show that $\widehat{\alpha}$ and $\widehat{\beta}$ are strongly consistent for α and β as $n \to \infty$.
2. Find the asymptotic distribution of $\widehat{\alpha}$ and $\widehat{\beta}$.

Part 2

Next assume that the time series X_t is generated by the model

$$X_t = \alpha + \frac{\beta}{\log t} + u_t, \quad t = 3, ..., n \tag{2}$$

3. Show that $\widehat{\alpha}$ and $\widehat{\beta}$ are strongly consistent for α and β as $n \to \infty$.
4. Find the asymptotic distribution of $\widehat{\alpha}$ and $\widehat{\beta}$.

Part 3

Suppose that u_t in (1) and (2) is the linear process

$$u_t = \sum_{i=0}^{\infty} c_i \varepsilon_{t-i}, \quad \text{with} \ \sum_{j=0}^{\infty} j\,|c_j| < \infty, \tag{3}$$

and where ε_t is *iid* $(0, \sigma^2)$ with finite fourth moment. Explain how you would modify your derivations in Parts 1 and 2 to allow for such an error process in the regression models (1) and (2).

Question B.

The time series X_t is generated by a components model of the form

$$X_t = g_t + X_t^0,\ t = 1, ..., n \tag{4}$$

where g_t is a deterministic trend function and X_t^0 is a fractional process satisfying

$$(1-L)^d X_t^0 = u_t, \quad t = 1, ..., n. \tag{5}$$

Here, n is the sample size, d is the 'memory' parameter and u_t satisfies (3) and is therefore stationary with mean zero and continuous spectral density $f_u(\lambda) = \frac{\sigma^2}{2\pi}\left|C(e^{i\lambda})\right|^2$.

In what follows, we define the discrete Fourier transform (dft) of a time series a_t by $w_a(\lambda) = (2\pi n)^{-1/2}\sum_{t=1}^{n} e^{i\lambda t}a_t$, the periodogram of a_t by $I_a(\lambda) = |w_a(\lambda)|^2$, and the 'fundamental' frequencies by $\lambda_s = \frac{2\pi s}{n}$, for $s = 0, 1, ...n-1$. We further define the 'modified' dft of a_t by

$$v_a(\lambda) = w_a(\lambda) + \frac{e^{i\lambda}}{1-e^{i\lambda}}\frac{a_n}{\sqrt{2\pi n}}, \tag{6}$$

for $\lambda \neq 0$, and the modified periodogram by $I_{v,a}(\lambda) = |v_a(\lambda)|^2$.

Part 1

1. Find formulae for $w_g(\lambda_s)$ when g_t has the explicit forms

$$g_t = \beta t, \tag{7}$$

and

$$g_t = \alpha + \beta t, \tag{8}$$

for some constant parameters α and β, and $\lambda_s = \frac{2\pi s}{n}$, $s = 0, 1, ...n-1$ are the fundamental frequencies.

2. Show that the modified transform $\{v_X(\lambda_s) : s = 1, ..., n-1\}$ is invariant to β but not α when g_t is given by (7) and (8). Suggest a modification to the definition of the modified transform (6) which achieves invariance to both α and β in the case of (8).

Part 2

Suppose g_t is given by (7). It is proposed to estimate the memory parameter d by log periodogram (LP) regression, LP regression with detrended data and modified LP regression. That is, the following linear least squares regressions are calculated:

$$\log I_X(\lambda_s) = \widehat{c} - 2\widehat{d}\log\lambda_s + \text{residual}, \quad (s = 1, ..., m), \tag{9}$$

$$\log I_{\widehat{X}^0}(\lambda_s) = c^* - 2d^*\log\lambda_s + \text{residual}, \quad (s = 1, ..., m), \tag{10}$$

$$\log I_{v,X}(\lambda_s) = \widetilde{c} - 2\widetilde{d}\log\lambda_s + \text{residual}, \quad (s = 1, ..., m), \tag{11}$$

where $\widehat{X}^0$ in (10) is the residual from a linear least squares regression of X_t on a linear trend, i.e.

$$\widehat{X}_t^0 = X_t - \widehat{\alpha} - \widehat{\beta}t, \ t = 1, ..., n.$$

In (9) - (11) the number of frequency ordinates used in the regression is $m \leq n$ and the ordinate corrresponding to $s = 0$ is excluded.

3. Are the estimates from the regressions (9) - (11) invariant to the values taken by the parameters α and β? Explain.

4. Compare the properties of the three regression estimates of d in a simulation study with $u_t \equiv iid\ N(0, 1)$ and using $m = [n^{2/3}]$, the integer part of $n^{2/3}$. You may generate data for X_t from (4) and (5), using the moving average representation

$$X_t^0 = \sum_{k=0}^{t} \frac{(d)_k}{k!} u_{t-k}, \ \ (d)_k = (d)(d+1)\ldots(d+k-1),$$

   and setting $u_j = 0$ for all $j \leq 0$. Try several values of d (e.g, $d = 0.3, 0.9, 1.5$) and β (e.g, $\beta = 0, 0.25$) in your simulations.

5. Discuss your findings.

Part 3

Briefly consider what happens to your answers in Part 1 and Part 2 when $g_t = \alpha + \beta t + \gamma t^2$ instead of (7) or (8).

Question C.

Download the Nelson Plosser (1982) macroeconomic data set and extended Nelson Plosser data (Schotman and van Dijk, 1991) from my web site at URL: http://korora.econ.yale.edu . The data are annual historical economic time series for the US over the period 1860-1980.

Suppose each of these series (X_t, say) is generated by a model of the form (4) and (7).

1. Estimate the memory parameter d for each of the series you have collected by LP regression, detrended LP regression and modified LP regression. That is, run each of the three regressions (9) - (11) with this data set. Compare the estimates.

2. You may assume that the usual asymptotic theory for linear regression applies in the case of the modified LP regression. Use this asymptotic theory to construct 95% confidence intervals for the memory parameter d for each of the series.

3. Discuss your results and their implications for the presence or absence of nonstationary long memory for these economic time series.

4. Can you test for unit root behavior using this approach?

Write your answers to this question as a short scientific paper, paying attention to the quality of your presentation. Be sure to indicate the limitations of the approach you are taking wherever you think it is appropriate to do so.

References

Nelson, C. R. and C. Plosser (1982). "Trends and random walks in macroeconomic time series: Some evidence and implications," *Journal of Monetary Economics* 10, 139–162.

Schotman, P. and H. K. van Dijk (1991). "On Bayesian routes to unit roots," *Journal of Applied Econometrics*, 6, 387-402