Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer ALL[1] Questions: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 9 January 1998, 12:00 noon.

[1] In the event that you are unable to complete all your answers in the allotted time, please be sure to attempt Questions A and D.

Question A.

Data $X^n = (X_t)_1^n$ is generated from the model

$$X_t = \varepsilon_t + \theta \varepsilon_{t-1}; \quad t = 1, \ldots, n \tag{1}$$

where $(\varepsilon_t)_1^n \sim \text{iid}\, N(0, \sigma^2)$; $\varepsilon_0 = 0$ and $|\theta| < 1$. In place of (1), the following autoregression is estimated by least squares regression with the data X^n

$$X_t = \hat{\alpha} X_{t-1} + \hat{u}_t. \tag{2}$$

1. Show that
$$\hat{\alpha} \to_{a.s.} m(\theta)$$
as $n \to \infty$ and find the limit function $m(\theta)$.

2. Derive the asymptotic distribution of $\hat{\alpha}$.

3. It is proposed to estimate θ by inverting the limit function $m(\theta)$, leading to
$$\hat{\theta} = m^{-1}(\hat{\alpha}). \tag{3}$$
Is this a feasible procedure? Will the estimator $\hat{\theta}$ be consistent for θ? Find the asymptotic distribution of $\hat{\theta}$.

4. Compare the asymptotic distribution of $\hat{\theta}$ with that of the maximum likelihood estimator of θ from (1).

5. Suppose that the following $AR(p)$ model is estimated in place of (2)
$$X_t = \hat{\alpha}_1 X_{t-1} + \ldots + \hat{\alpha}_p X_{t-p} + \hat{u}_t.$$
Show that
$$\hat{\alpha} \to_{a.s.} m(\theta)$$
as $n \to \infty$, where $\hat{\alpha} = (\hat{\alpha}_1, \ldots, \hat{\alpha}_p)'$, and find the vector limit function $m(\theta)$. It is now proposed to estimate θ by minimising
$$\hat{\theta} = \arg\min_{\theta} (\hat{\alpha} - m(\theta))'(\hat{\alpha} - m(\theta)). \tag{4}$$
Examine the asymptotic properties of $\hat{\theta}$ in this case.

6. Do a simulation study to compare the finite sample behaviour of these autoregressive estimators of θ for $p = 1, 2, 3$.

Question B.

Given a time series $X^n = (X_t)_1^n$; define the discrete Fourier transform $w_x(\lambda) = n^{-1/2}\sum_{t=1}^{n} e^{i\lambda t} X_t$. Let n be even and introduce integers m and M for which $n = 2mM$. It is assumed that $m, M \to \infty$ and $\frac{m}{n} + \frac{M}{n} \to 0$ as $n \to \infty$. Define $\omega_j = \frac{\pi j}{M}$; for $j = -M + 1, \ldots, M - 1$ and define the intervals

$$B_j = \left(\omega_j - \frac{\pi}{2M} < \lambda \leq \omega_j + \frac{\pi}{2M} \right).$$

1. Find a formula for $w_d(\lambda_s)$ when $d_t = \frac{t}{n}$ and $\lambda_s = \frac{2\pi s}{n}$; $s = 0, 1, \ldots n$.

2. Discuss the limit behaviour of $w_d(\lambda_s)$ as $n \to \infty$.

3. Find a normalizing quantity for and derive the limit behaviour of the smoothed periodogram $\sum_{\lambda_s \in B_0} w_d(\lambda_s) w_d(\lambda_s)^*$ of $w_d(\lambda_s)$ over the interval B_0.

4. Now consider the regression model

$$y_t = b\frac{t}{n} + u_t \tag{5}$$

   where u_t is stationary with zero mean and continuous spectrum $f_{uu}(\lambda) > 0$. It is proposed to estimate the trend coefficient b in (5) by the spectral regression estimator

$$\hat{b} = \frac{\sum_{\lambda_s \in B_0} w_y(\lambda_s) w_d(\lambda_s)^*}{\sum_{\lambda_s \in B_0} w_d(\lambda_s) w_d(\lambda_s)^*}.$$

   Find the asymptotic distribution of $\hat{b}$ as $n \to \infty$.

5. Is $\hat{b}$ asymptotically efficient in some sense? Explain.

Question C.

Consider the model

$$y_t = \beta' x_t + \varepsilon_t; \quad t = 1, \ldots, n \tag{6}$$

where the errors ε_t are iid $N(0, \sigma^2)$ and x_t is a 2×1 vector of exogenous variables satisfying

$$x_t = \gamma \frac{1}{t^a} + \mu v_t$$

for some $a \in (0, \frac{1}{2})$ and where the vectors γ and μ are linearly independent and the v_t are iid $N(0, \sigma_v^2)$ and independent of the ε_t.

1. Find the limit behaviour of the least squares estimate $\hat{\beta}$ of β in (6).
2. What happens to this limit theory as $a \to \frac{1}{2}$?
3. Is the usual least squares asymptotic theory of inference valid?

Question D[2]

1. Download multi-country macro data files from the Penn World Tables at the Toronto or NBER sites on the internet. Below is the Toronto address. Just follow the instructions for downloading files given at this site. The URL is http://datacentre.epas.utoronto.ca:5680/pwt/pwt.html
2. The data set provides annual data for 152 countries. Select the series for real GDP per capita (RGDPC) for 5 countries, including the USA. Make sure that you have at least 30 observations for each series. With the data you have obtained in this way, perform the following empirical exercise whose object is to estimate the growth rate of real GDP per capita for each country.

   (a) Use the simple trend regression model

$$y_t = a + bt + u_t; \quad t = 1, \ldots, n \tag{7}$$

[2] This empirical exercise is based the recent cross country study by Canjels and Watson (1997) on growth rate estimation. See also the paper by Phillips and Lee (1996) on efficient trend extraction.

where y_t is the logarithm of real per capita GDP and u_t is an error process which, in general, is serially correlated. Assume that

$$u_t = \alpha u_{t-1} + \varepsilon_t \qquad (8)$$

where ε_t is iid $(0; \sigma^2)$:

(b) Estimate equation (7) under assumption (8) and report your estimates of the growth rate parameter . Try using the following feasible procedures. Report your estimates in a Table so that it is easy for you to consider and comment on your results.

   i. Simple OLS regression of (7);

   ii. Cochrane-Orcutt regression of (7) with $n - 1$ transformed observations using an estimate of α in (8) obtained from a preliminary regression that uses OLS regression residuals. Report your estimates of α;

   iii. Cochrane-Orcutt estimation of (7) supplemented with the following equation for the first observation

   $$y_1 = const. + u_1:$$

   That is, use feasible generalised least squares estimation of (7) using n observations.

   iv. Simple OLS regression on (7) reformulated in first differences.

(c) Replace (8) with a general AR(p) model for the errors and use the BIC model selection criterion to estimate the order, p, of this autoregression using the residuals from an OLS regression on (7) as data. Use the fitted AR(p) model to transform the data in (7) and then reestimate the parameter . Report the new estimates alongside those obtained by the other methods.

(d) Briefly discuss your results and compare them with those of Canjels and Watson (1997).

References[3]

Canjels, N. And M. Watson (1997). "Estimating deterministic trend in the presence of serially correlated errors," Review of Economics and Statistics, 184-200.

Phillips, P.C.B., and C. C. Lee (1996), "Efficiency Gains from Quasi-Differencing under Nonstationarity" in P.M. Robinson and M. Rosenblatt (Eds.) "Essays in Memory of E.J. Hannan". New York: Springer.

[3] Copies of these two references are available from Elizabeth in 30 Hillhouse Avenue.