Econ. 553a
YaleUniversity

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Fall 1996

# Econometrics IV: Time Series Econometrics

# Take Home Examination

*Answer ALL Questions*: Any reference material allowed
*Time Allowed*: Four weeks
*Due Date & Time*: Monday 16 December, 12:00 noon.

**Question 1.**

Consider trend regression model

$$y_t = bt + u_t, \quad t = 1, ..., n \tag{1}$$
$$u_t = \varepsilon_t - \theta\varepsilon_{t-1} \tag{2}$$

where $(\varepsilon_t)_1^n \equiv iidN(0, \sigma^2)$ and $\varepsilon_0 = 0$. Let $\widehat{b}$ be the OLS estimate and $\widetilde{b}$ be the GLS estimate of $b$ in $(2)$.

(a) If $\theta = 1$ in $(2)$, find the asymptotic distributions of $\widehat{b}$ and $\widetilde{b}$. Compare and discuss your results. Does the Grenander–Rosenblatt (1957) theorem hold? Explain.

(b)*[1] Suppose $\theta = 1 + \frac{c}{n}$, for some constant $c$. Find the limit distributions of $\widehat{b}$ and $\widetilde{b}$ in this case.

(c)* Can you find limits for thelimit distributions in part (b) as $c \to -\infty$? Does the Grenander–Rosenblatt theorem apply to the new limits? Explain.

**Question 2.**

Consider the following representation of a standard Brownian motion $W(r)$ on the interval $[0, 2\pi]$

$$W(r) = \frac{r}{\sqrt{\pi}}X_0 + \sqrt{\frac{2}{\pi}}\sum_{m=1}^{\infty}\frac{\sin mr}{m}X_m,$$

where the $X_m$ are $iidN(0, 1)$. Prove that this series converges almost surely for $r \in [0, 2\pi]$.

**Question 3.**

Consider the model

$$y_t = \mu A i + A x_t + \epsilon_t, \quad t = 1, ..., n \tag{3}$$

where $y_t$ is an $m-$ vector of dependent variables, $x_t$ is a $k-$ vector of strictly stationary exogenous variables and the errors $\epsilon_t$ are $iidN(0, \Omega)$ with $\Omega$ positive definite. The coefficient matrix $A$ in (3)is unrestricted, $i$ is a sum vector (i.e. a $k-$ vector with unity in each element) and $\mu$ is a scalar parameter.

(a) Set up a likelihood function for model (3) and derive formulae for the maximum likelihood estimates $\left(\widehat{\mu}, \widehat{A}, \widehat{\Omega}\right)$of the parameters $\mu$, $A$ and $\Omega$.

[1] The asterisked sections are more difficult. Do not be concerned if you find these sections too hard to complete. Just give them your best attempt without spending an enormous amount of time on them.

(b) Show that $\left(\widehat{\mu}, \widehat{A}, \widehat{\Omega}\right) \overset{a.s}{\to} (\mu, A, \Omega)$.

(c) Find the limit distribution of $\sqrt{n}\,(\widehat{\mu} - \mu)$ and give an explicit formula for its limiting variance.

(d) Take the special case where $\Omega = \sigma^2 I$. Compare the limit variance of $\sqrt{n}\,(\widehat{\mu} - \mu)$ with that of $\sqrt{n}\,(\widetilde{\mu} - \mu,)$ where $\widetilde{\mu}$ is the single equation estimator of $\mu$ based on any one of the equations in (3).

(e)Find the limit distribution of $\sqrt{n}\left(\widehat{A} - A\right)$. Compare your results in the two cases $\mu = 0$, and $\mu \neq 0$ and explain the differences.

(e) Indicate how you would use the limit distributions to find a confidence interval for $\mu$.

**Question 4.** (*Empirical Application of Question 3*)

Staiger Stock and Watson (1996a,1996b) recently obtained empirical estimates of the NAIRU (non-accelerating inflation rate of unemployment) for the USA from single equation regressions involving both quarterly and monthly data of price inflation ($\pi_t$), unemployment ($u_t$) and several additional variables ($X_t$). The equation these authors estimated has the form

$$\Delta\pi_t = \eta_\pi + \beta(L)\, u_{t-1} + \gamma(L)' X_t + \varepsilon_{\pi t} \tag{4}$$

in which the NAIRU is specified as the function

$$\mu = -\eta_\pi / \beta(1). \tag{5}$$

Consider a two equation version of model (4)that involves wage inflation ($\omega_t$) as a second dependent variable, i.e.

$$\Delta\omega_t = \eta_\omega + \alpha(L)\, u_{t-1} + \delta(L)' X_t + \varepsilon_{\omega t} \tag{6}$$

in which the NAIRU has the form

$$\mu = -\eta_\omega / \alpha(1). \tag{7}$$

(a) Write equations $(4 - 7)$ as a system of two equations with a restricted intercept in the same general form as model (3) in Question 3.

(b) Estimate the NAIRU from this two equation system using the maximum likelihood estimator as in Question 3. Use quarterly or monthly postwar data (or both) from the Citibase data set[2]. You may try some alternative specifications that involve different covariates $X_t$ in the equations.

[2] I have left a diskette with the data that Staiger, Stock and Watson used with Elizabeth in 30 Hillhouse Avenue. Please copy this diskette for your own work and return to Elizabeth.

(c) Find confidence intervals for your multiple equation NAIRU estimates using the asymptotic theory in Question 3. Compare the precision of your interval estimates with those reported in the Staiger et al (1996a &b) papers for equivalent specifications. Is there any gain from the use of multiple equation estimation?

## References[3]

Staiger, D., J. H. Stock and M. W. Watson (1996a) "How precise are estimates of the natural rate of unemployment", in C. Romer and D. Romer (Eds.) *Monetary Policy and Low Inflation.* Chicago: University of Chicago Press. (to appear)

Staiger, D., J. H. Stock and M. W. Watson (1996b) "The NAIRU, Unemployment and Monetary Policy", *Journal of Economic Perspectives.* (to appear).

[3]Copies of these two references are available from Elizabeth in 30 Hillhouse Avenue.