

Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question B or Question C

Time Allowed: Eight weeks

Due Date & Time: Monday 1 January 2018, 5:00pm.

Electronic Filing: Submit your typed papers by email to: peter.phillips@yale.edu

References: Any reference material is allowed.

Question A (Asymptotic Theory in a Nonlinear Autoregression)

In the stationary nonlinear time series model

$$y_t = \theta y_{t-1} + \alpha s_{t-1}^2 + u_t, \quad t = 1, \dots, n, \quad (1)$$

$$s_t = v_t + \beta v_{t-1}, \quad (2)$$

$$|\theta| < 1, \quad |\beta| < 1$$

the variable y_t is observable and data $\{y_t\}_{t=0}^n$ are available for estimation. The variable s_t is an unobserved state variable and appears nonlinearly as a quadratic component of (1). The equation error $u_t \sim iid(0, \mathbb{E}(u_t^2) = \sigma_u^2)$ and is independent of the state variable innovations $v_t \sim iid(0, \mathbb{E}(v_t^2) = \sigma_v^2, \mathbb{E}(v_t^4) = \mu_4')$. The stationary process y_t generated by the above system is assumed to have initial conditions in the infinite past.

Using the observed data $\{y_t\}_{t=0}^n$, it is proposed to estimate the autoregressive coefficient θ by fitting a least squares autoregression with intercept giving

$$y_t = \hat{\gamma} + \hat{\theta} y_{t-1} + \hat{u}_t. \quad (3)$$

- (i) Find the probability limit of the estimator $\hat{\theta}$. Under what special conditions is $\hat{\theta}$ consistent for θ ? Discuss your results.
- (ii) Show that a consistent estimator θ_{IV} of θ may be obtained by using y_{t-p} , with $p \geq 2$, as an instrument for y_{t-1} in an instrumental variable (IV) form of the autoregression (3).
- (iii) Making any additional assumptions you may need, find the asymptotic distribution of the IV estimator suggested in part (ii).
- (iv) Perform a simulation exercise, showing the finite sample performance of $\hat{\theta}$ and the instrumental variable estimator θ_{IV} for a selection of values of the parameters $(\theta, \alpha, \beta, \sigma_u^2, \sigma_v^2)$.
- (v) Suppose $\theta = 1$ in the model (1) and that the process is initiated at $t = 0$ from the initial condition $y_0 = 0$. Making any further assumptions that may be required, find the asymptotic behavior of the estimator $\hat{\theta}$ in this case. Discuss your results.

Question B (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evaluation of the research direction, including its strengths and limitations. A full set of references should be included, with an indication of the key source material that has been used in writing this paper.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and empirical findings as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to or used as your Applied Econometrics Paper for the departmental requirement.