Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question B or Question C

Time Allowed: Nine weeks
Due Date & Time: Monday 16 January 2017, 5:00pm.
Electronic Filing: Submit your typed papers by email to: peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Wald Test of a Nonlinear Stochastic Trend)

An empirical investigator is working with time series data $\{X_t\}_{t=0}^n$. Theory suggests the null hypothesis of a stochastic trend model

$$X_t = X_{t-1} + u_t, \quad t = 1, ..., n. \tag{1}$$

The investigator suspects nonlinear effects are present in the data and decides to run the empirical least squares regression

$$X_t = \hat{\theta} X_{t-1} + \hat{\beta}' Z_{t-1} + \hat{u}_t, \tag{2}$$

where $Z'_{t-1} = \left(X^2_{t-1}, X^3_{t-1}, ..., X^{p+1}_{t-1}\right)$. To test for the presence of significant effects from Z_{t-1} on X_t, the investigator decides to use a Wald test based on the usual statistic

$$W_\beta = \hat{\beta}' \left(Z'_{-1} Q_{X_{-1}} Z_{-1}\right) \hat{\beta} / \hat{\sigma}^2,$$

where $\hat{\sigma}^2 = n^{-1} \sum_{t=1}^n \hat{u}_t^2$, $Z_{-1} = [Z_0, Z_1, ..., Z_{n-1}]'$, $X_{-1} = [X_0, X_1, ..., X_{n-1}]'$, and $Q_{X_{-1}} = I_n - X_{-1} \left(X'_{-1} X_{-1}\right)^{-1} X'_{-1}$.

Part A Assume that $u_t \sim_{iid} (0, \sigma^2)$, $X_0 = O_p(1)$, and the null model (1) is correctly specified.

(i) Find the asymptotic distributions of the estimator $\hat{\beta}$ and the Wald statistic W_β.

(ii) Setting $p = 2$, perform a simulation study to examine the finite sample distributions of $\hat{\beta}$ and W_β for various values of n.

Part B Assume that the null model (1) still holds but that $u_t = \sum_{j=0}^\infty c_j \varepsilon_{t-j}$, where $\sum_{j=0}^\infty j |c_j| < \infty$, $c(1) = \sum_{j=0}^\infty c_j \neq 0$, and $\varepsilon_t \sim_{iid} (0, \sigma_\varepsilon^2)$. Define $\omega^2 = \sigma_\varepsilon^2 c(1)^2$, the long run variance of u_t, and $\sigma^2 = \sum_{j=0}^\infty c_j^2 \sigma_\varepsilon^2$, the variance of u_t.

(iii) Making any other assumptions you may need, find the asymptotic distributions of the estimator $\hat{\beta}$ and the Wald statistic W_β. Compare your results with those you have obtained in Part A(i). Show that the statistic W_β is no longer asymptotically pivotal.

(iv) Suggest a modified form of Wald statistic that is asymptotically pivotal and prove your results.

Part C

(v) Making any additional assumptions you may need, repeat **Part A** for the case where the fitted regression (2) involves the regressor

$$Z'_{t-1} = \left(X^2_{t-1}\left(\Delta X_{t-1}\right)^2, X^3_{t-1}\left(\Delta X_{t-1}\right)^3, ..., X^{p+1}_{t-1}\left(\Delta X_{t-1}\right)^{p+1}\right).$$

Question B (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations. A full set of references should be included, with an indication of the key source material that has been used in writing this paper.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and empirical findings as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to or used as your Applied Econometrics Paper for the departmental requirement.