Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question B or Question C

Time Allowed: Nine weeks
Due Date & Time: Friday 8 January 2016, 5:00pm.
Electronic Filing: Submit your typed papers by email to: peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Prediction with Partial Information)

The linear structural equation $y_{1t} = y_{2t}\beta + u_t$ relates two observed endogenous variables (y_{1t}, y_{2t}) and has unknown structural coefficient β and equation error u_t. In observation format, the structural equation is written as

$$y_1 = y_2\beta + u, \tag{1}$$

with $y_1 = [y_{1,1}, ...y_{1,n}]'$ and $y_2 = [y_{2,1}, ...y_{2,n}]'$. The associated reduced form is written

$$[y_1, y_2] = Z[\pi_1, \pi_2] + [v_1, v_2] = Z\Pi + V, \tag{2}$$

with $n \times K$ observation matrix $Z = [z_1, ..., z_n]'$ of $K \geq 1$ observed exogenous variables z_t that are used to instrument $y_{2,t}$ in (1). The $K \times 2$ reduced form coefficient matrix $\Pi = [\pi_1, \pi_2]$ is partitioned conformably with $[y_1, y_2]$, as is the reduced form error matrix $V = [v_1, v_2]$, which is assumed to be normally distributed. The coefficients of the structural form and reduced form are related by the equation $\pi_1 = \pi_2\beta$. Standardizing transformations (see Phillips, 1980, 1983) are applied (with no loss of generality) to (2) so that $Z'Z = nI_K$ and $V \sim_d N(0, I_{2n})$.

The parameter β in (1) is estimated by instrumental variables leading to $\tilde{\beta} = (y_2'P_Zy_2)^{-1}(y_2'P_Zy_1)$, where $P_Z = Z(Z'Z)^{-1}Z'$. The reduced form coefficient vector π_1 is estimated by unrestricted reduced form least squares giving $\hat{\pi}_1 = (Z'Z)^{-1}Z'y_1$ and by partially restricted reduced form estimation giving $\tilde{\pi}_1 = \hat{\pi}_2\tilde{\beta}$, where $\hat{\pi}_2 = (Z'Z)^{-1}Z'y_2$.

It is proposed to predict the next period observation $y_{1,n+1} = z_{n+1}'\pi_1 + v_{1,n+1}$ by using these two reduced form estimates, giving the two predictors $\hat{y}_{1,n+1} = z_{n+1}'\hat{\pi}_1$ and $\tilde{y}_{1,n+1} = z_{n+1}'\tilde{\pi}_1$. It is assumed that z_{n+1} is observed and equal to $z_{n+1} = \iota_K/\sqrt{K}$, where $\iota_K' = (1, ..., 1)$ is a K-vector of ones. The next period error $v_{1,n+1} \sim_d N(0, 1)$ and is independent of V.

In the questions below, assume that the true value of the reduced form coefficient matrix $\Pi = 0$, so that the structural coefficient β is only apparently identified by the order condition $K \geq 1$.

1. Find the exact finite sample distribution of the two reduced form predictors $\hat{y}_{1,n+1}$ and $\tilde{y}_{1,n+1}$.

2. Find the forecast mean squared errors (FMSE) $\mathbb{E}(\hat{y}_{1,n+1} - y_{1,n+1})^2$ and $\mathbb{E}(\tilde{y}_{1,n+1} - y_{1,n+1})^2$.

3. Compare the behavior of the predictors and FMSEs as K increases.

4. Compute the predictive densities of $\hat{y}_{1,n+1}$ and $\tilde{y}_{1,n+1}$ for various values of K. You may use the exact density formulae from part (1) or simulation methods and kernel density estimation if you have not obtained formulae for the exact densities in part (1).

5. Indicate how your results apply when the Gaussian assumption $V \sim_d N(0, I_{2n})$ is relaxed and central limit theory is used to deliver a limit distribution theory.

6. Discuss your findings and the role of partial information in prediction.

Question B (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.

Reference

Phillips, P. C. B. (1980). "The exact finite sample density of instrumental variable estimators in an equation with $n+1$ endogenous variables," *Econometrica* 48:4, 861–878.

Phillips, P. C. B. (1983). "Exact small sample theory in the simultaneous equations model," Chapter 8 and pp. 449–516 in M. D. Intriligator and Z. Griliches (eds.), *Handbook of Econometrics*. Amsterdam: North–Holland.