Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question B or Question C

Time Allowed: Five weeks
Due Date & Time: Monday 29 December 2014, 5:00pm.
Electronic Filing: Submit your typed papers by email to: peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Inference in Near Singular Regression)

In the model

$$y_t = x_t'\beta + u_{0t}, \quad t = 1, ..., n \tag{1}$$

the errors $u_{0t} \sim iid\,(0, \sigma_{00})$ and the regressors

$$x_t = \begin{bmatrix} x_t^0 \\ \gamma' x_t^0 + \frac{1}{t} \end{bmatrix},$$

where $x_t^0 \sim iid\,(0, \Sigma_{xx})$ is an $m \times 1$ vector independent of u_{0s} for all (s, t), and $\gamma \neq 0$ is an $m \times 1$ vector of unknown constants.

The $(m + 1) \times 1$ parameter vector β in (1) is estimated by ordinary least squares regression using $\hat{\beta} = (X'X)^{-1} X'y$ where $X' = [x_1, ..., x_n]$ and $y' = [y_1, ..., y_n]$. It is proposed to test the null hypothesis $\mathbb{H}_0 : \beta = 0$ using the Wald test statistic $W_n = \hat{\beta}' X'X\hat{\beta}/\hat{\sigma}^2$, where $\hat{\sigma}^2 = n^{-1}y'\left(I_n - X(X'X)^{-1}X\right)y$ is the usual sample variance of the regression residuals.

Part (I): Find the limit behavior (including the limit distributions) of $\hat{\beta}$ and W_n as $n \to \infty$. Discuss your findings.

Part (II): Suppose x_t is endogenous and it is proposed to estimate (1) by instrumental variable regression using $\tilde{\beta} = (X'P_z X)^{-1} (X'P_z y)$ where $P_z = Z(Z'Z)^{-1}Z$, $Z' = [z_1, ..., z_n]$ and $z \sim iid\,(0, \Sigma_{zz})$ is a $K \times 1$ vector of instruments $(K \geq m + 1)$. It is assumed that the vector $(u_{0t}, x_t^{0\prime}, z_t') \sim iid\,(0, \Sigma)$ with positive definite covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{00} & \sigma_{0x} & 0 \\ \sigma_{x0} & \Sigma_{xx} & \Sigma_{xz} \\ 0 & \Sigma_{zx} & \Sigma_{zz} \end{bmatrix}$$

and that the submatrix $\Sigma_{xz} = \mathbb{E}\left(x_t^0 z_t'\right)$ has full rank m.

(a) Discuss conditions for the validity of the orthogonality and relevance of the instruments z_t for x_t.

(b) Find the limit behavior of $\tilde{\beta}$ when $n \to \infty$.

(c) Find the limit behavior of the IV Wald test $\tilde{W}_n = \tilde{\beta}' X'P_Z X\tilde{\beta}/\tilde{\sigma}^2$ of $\mathbb{H}_0 : \beta = 0$ where $\tilde{\sigma}^2 = \frac{1}{n}\sum_{t=1}^n \tilde{u}_{0t}^2$ is the usual variance estimate based on the IV residual $\tilde{u}_{0t} = y_t - x_t'\tilde{\beta}$.

(d) Find the limit behavior of the overidentification test statistic $S_n = \tilde{u}_0' P_z \tilde{u}_0 / \tilde{\sigma}^2$ where $\tilde{u}_0 = (\tilde{u}_{01}, ..., \tilde{u}_{0n})'$.

(e) Discuss your findings.

Question B (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.