Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question C or Question D

If you want to take on a special challenge in the mysteries of econometrics, try Question B as well.

Time Allowed: Six weeks
Due Date & Time: Friday 3 January 2014, 5:00pm.
Electronic Filing: Submit your typed papers by email to peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Partially Unidentified GMM Panel Regression)

In the dynamic panel regression model

$$y_{it} = \alpha_i (1-\rho) + \rho y_{it-1} + u_{it}, \quad i = 1, ..., n; t = 1, ...T \tag{1}$$

the α_i are fixed effects, $y_{i0} = 0$ for all i, and the errors $u_{it} \sim iid\ N(0, \sigma^2)$ across i and over t. It is proposed to difference (1) to eliminate the fixed effects and then to estimate the autoregressive coefficient ρ using GMM with instrument y_{it-2} leading to the estimator

$$\rho_{gmm} = \frac{\sum_{i=1}^{n}\sum_{t=2}^{T} \Delta y_{it} y_{it-2}}{\sum_{i=1}^{n}\sum_{t=2}^{T} \Delta y_{it-1} y_{it-2}}.$$

The true value of the autoregressive coefficient is $\rho = 1$.

(i) Discuss the validity of the instrumental variable y_{it-2} for estimating ρ.

(ii) Find the limit behavior (including the limit distribution) of ρ_{gmm} under the following conditions:

- **(a)** For fixed T as $n \to \infty$. Then, find the sequential limit behavior when $n \to \infty$ is followed by $T \to \infty$.
- **(b)** For fixed n as $T \to \infty$. Then, find the sequential limit behavior when $T \to \infty$ is followed by $n \to \infty$.

(iii) What can you say about joint convergence when $(n, T) \to \infty$ jointly?

(iv) Discuss your findings.

Question B (Singular Cointegrating Regression)

In the cointegrating regression model

$$y_t = Ax_t + u_{0t}, \tag{2}$$
$$x_t = x_{t-1} + u_{xt}, \quad t = 1, ..., n \tag{3}$$

A is an $m_0 \times m_x$ coefficient matrix, x_t is initialized at $t = 0$ by $x_0 = O_p(1)$, and the combined error vector $u_t = (u_{0t}', u_{xt}')'$ follows the linear process

$$u_t = D(L)\eta_t = \sum_{j=0}^{\infty} D_j \eta_{t-j}, \text{ with } \sum_{j=0}^{\infty} j \|D_j\| < \infty, \ \eta_t \sim iidN(0, I_m),$$

where $m = m_0 + m_x$. The linear operator $D(L)$ and long run variance matrix $\Omega = D(1) D(1)'$ of u_t are partitioned conformably with u_t as

$$D(L) = \begin{bmatrix} D_{00}(L) & D_{0x}(L) \\ D_{x0}(L) & D_{xx}(L) \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_{00} & \Omega_{0x} \\ \Omega_{x0} & \Omega_{xx} \end{bmatrix},$$

where $\Omega_{xx} > 0$ is positive definite. The conditional long run covariance matrix $\Omega_{00.x} = \Omega_{00} - \Omega_{0x}\Omega_{xx}^{-1}\Omega_{x0}$ is singular and has representation $\Omega_{00.x} = RR'$ where R is $m_0 \times r$ of rank $r < m_0$.

It is proposed to estimate the coefficient matrix A in (2) by fully modified least squares (FMOLS).

(i) Find the limit distribution of the FMOLS estimator $\hat{A}^+$ and the rates of convergence of its elements.

(ii) Explain how your result compares with the usual case where $\Omega_{00.x}$ is positive definite.

Question C (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question D (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.