Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question C or Question D

If you want to further confront the mysteries of econometrics, then try Question A AND Question B.

Time Allowed: Six weeks
Due Date & Time: Sunday 15 January 2012.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Inference in Trend Regression)

The time series $\{X_t\}_{t=1}^n$ is generated by the mechanism

$$\mathbb{H}_0 : X_t = at + X_t^0, \quad \text{with } a = 0,$$

where $X_t^0 = \sum_{s=1}^t u_s$ is a partial sum of a zero mean stationary process u_s with continuous spectral density $f_u(\lambda)$ and satisfies the functional law

$$X_n(r) = n^{-1/2} X_{\lfloor nr \rfloor}^0 \Rightarrow B(r) \equiv BM\left(\omega^2\right), \quad \text{where } \omega^2 = 2\pi f_u(0) > 0. \tag{1}$$

Using the data $\{X_t\}_{t=1}^n$, an investigator fits by least squares regression the trend regression model

$$X_t = \hat{a}t + \hat{v}_t \tag{2}$$

and proposes to test the significance of $\hat{a}$ using the following t-ratio statistics:

$$t_a = \frac{\hat{a}}{s_a}, \quad t_a^{hac} = \frac{\hat{a}}{\hat{\omega}_a}, \quad t_a^{har} = \frac{\hat{a}}{\breve{\omega}_a}. \tag{3}$$

In (3) the standardizations are defined by the quantities

$$\begin{aligned}
s_a^2 &= \frac{s_v^2}{\sum_{t=1}^n t^2}, \quad s_v^2 = \frac{1}{n}\sum_{t=1}^n \hat{v}_t^2, \\
\hat{\omega}_a^2 &= \left(\sum_{t=1}^n t^2\right)^{-1} \left(n \text{ lvar}_{HAC}(t\hat{v}_t)\right) \left(\sum_{t=1}^n t^2\right)^{-1}, \\
\breve{\omega}_a^2 &= \left(\sum_{t=1}^n t^2\right)^{-1} \left(n \text{ lvar}_{HAR}(t\hat{v}_t)\right) \left(\sum_{t=1}^n t^2\right)^{-1},
\end{aligned}$$

where

$$\begin{aligned}
\text{lvar}_{HAC}(t\hat{v}_t) &= \sum_{j=-M}^{M} k\left(\frac{j}{M}\right) C(j, t\hat{v}_t), \\
\text{lvar}_{HAR}(t\hat{v}_t) &= \sum_{j=-n+1}^{n-1} k_b\left(\frac{j}{n}\right) C(j, t\hat{v}_t),
\end{aligned}$$

in which M is a bandwidth (lag truncation) parameter satisfying $\frac{1}{M}+\frac{M}{n}\to 0$ as $n\to\infty$,

$$C(j,Z_t)=\frac{1}{n}\sum_{1\le t,t+j\le n} Z_t Z_{t+j},$$

$k(x)$ is an integrable lag kernel function and $k_b(x)=k\left(\frac{x}{b}\right)$ for some fixed constant $b\in(0,1]$.

(i) Find the limit behavior of t_a, t_a^{hac}, and t_a^{har} as $n\to\infty$ under the null hypothesis $\mathbb{H}_0$.

(ii) Now suppose that the data $\{X_t\}_{t=1}^n$ are generated by the alternative mechanism

$$\mathbb{H}_1 : X_t = at + X_t^0, \quad \text{with } a\neq 0.$$

Find the limit behavior of t_a, t_a^{hac}, and t_a^{har} as $n\to\infty$ under the alternative hypothesis $\mathbb{H}_1$.

(iii) Perform a simulation experiment to show finite sample performance of the tests.

(iv) Discuss your findings.

Question B (Singular Cointegrating Regression)

In the cointegrating regression model

$$y_t = Ax_t + u_{0t}, \tag{4}$$
$$x_t = x_{t-1} + u_{xt}, \quad t=1,...,n \tag{5}$$

A is an $m_0\times m_x$ coefficient matrix, x_t is initialized at $t=0$ by $x_0=O_p(1)$, and the combined error vector $u_t=(u_{0t}',u_{xt}')'$ follows the linear process

$$u_t = D(L)\eta_t=\sum_{j=0}^{\infty} D_j\eta_{t-j}, \text{ with } \sum_{j=0}^{\infty} j\|D_j\|<\infty, \ \eta_t\sim iidN(0,I_m),$$

where $m=m_0+m_x$. The linear operator $D(L)$ and long run variance matrix $\Omega=D(1)D(1)'$ of u_t are partitioned conformably with u_t as

$$D(L)=\begin{bmatrix} D_{00}(L) & D_{0x}(L)\\ D_{x0}(L) & D_{xx}(L)\end{bmatrix}, \quad \Omega=\begin{bmatrix}\Omega_{00} & \Omega_{0x}\\ \Omega_{x0} & \Omega_{xx}\end{bmatrix},$$

where $\Omega_{xx} > 0$ is positive definite. The conditional long run covariance matrix $\Omega_{00.x} = \Omega_{00} - \Omega_{0x}\Omega_{xx}^{-1}\Omega_{x0}$ is singular and has representation $\Omega_{00.x} = RR'$ where R is $m_0 \times r$ of rank $r < m_0$.

It is proposed to estimate the coefficient matrix A in (4) by fully modified least squares (FMOLS).

(i) Find the limit distribution of the FMOLS estimator $\hat{A}^+$ and the rates of convergence of its elements.

(ii) Explain how your result compares with the usual case where $\Omega_{00.x}$ is positive definite.

Question C (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question D (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.