Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question C or Question D

> But wait! Does econometrics entice you with its many mysteries? And do you seek a seriously demanding challenge? If so, then try Question A AND Question B.

Time Allowed: Four weeks
Due Date & Time: Tuesday 28 December 2010.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (VAR with Mixed Order Regressors)

The bivariate autoregression

$$X_t = R_n X_{t-1} + u_t, \quad t = 1, ..., n \tag{1}$$

models the observed time series $X_t = (X_{1t}, X_{2t})'$ from a zero initialization $X_0 = 0$ with innovations $u_t \sim iid\ (0, \Sigma)$ where Σ is positive definite. The coefficient matrix $R_n = (r_{ij})$ is unknown but its true form is given by the diagonal matrix

$$R_n = \begin{bmatrix} \rho_n & 0 \\ 0 & \theta_n \end{bmatrix}, \quad \text{with } \rho_n = 1 + \frac{c}{n}, \quad \theta_n = 1 + \frac{b}{k_n} \tag{2}$$

for some fixed constant c, fixed positive constant $b > 0$, and where the sequence k_n satisfies $\frac{1}{k_n} + \frac{k_n}{n} \to 0$ as $n \to \infty$.

(i) The model (1) is fitted by unrestricted least squares regression giving the matrix coefficient estimate $\hat{R}_n$ of R_n. Show that $\hat{R}_n$ is consistent and find its limit distribution.

(ii) Describe how you would test the hypothesis $H_0 : r_{11} = r_{22}$ using a standard Wald test. Find the limit behavior of this test statistic when R_n has the explicit form given in (2).

Question B (VAR with Common Mildly Explosive Regressors)

In the bivariate autoregression

$$X_t = R_n X_{t-1} + u_t, \quad t = 1, ..., n \tag{3}$$

$$R_n = \begin{bmatrix} \theta_n & 0 \\ 0 & \theta_n \end{bmatrix}, \quad \theta_n = 1 + \frac{b}{k_n}, \quad b > 0, \tag{4}$$

the error process u_t is $iid\ (0, \Sigma)$ with $\Sigma > 0$, the process is initialized at $t = 0$ by $X_0 = 0$, θ_n is a mildly explosive coefficient with $b > 0$ and sequence k_n satisfying $\frac{1}{k_n} + \frac{k_n}{n} \to 0$ as $n \to \infty$.

(i) Find the limit distribution of the unrestricted least squares estimator $\hat{R}_n$ of $R_n = (r_{ij})$ as $n \to \infty$.

(ii) Find the limit behavior of the Wald test statistic for the hypothesis $H_0 : r_{11} = r_{22}$ when R_n has the explicit form given in (4).

Question C (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question D (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.