Econ. 553a
Yale University

Peter C. B. Phillips
Fall 2009

Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question C or Question D

> Do you hunger for a greater challenge? Does econometrics hold you in its grip with a growing enchantment? And do you want to take up the calling? If so, then try Question A AND Question B.

Time Allowed: Six weeks
Due Date & Time: Monday 11 January 2010.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu
References: Any reference material is allowed.

Question A (Exponential Trend Regression)

A1 The trend regression equation

$$Y_t = \alpha\rho^t + u_t, \quad t = 1, ..., n \tag{1}$$

models the scalar observed time series Y_t in terms of the exponential trend ρ^t where $\rho > 1$ is known, where α is an unknown coefficient to be estimated, and where u_t is $iid\ N(0, \sigma^2)$ with $\sigma^2 > 0$.

(i) The model (1) is fitted by linear least squares regression giving the coefficient estimate $\hat{\alpha}$. Show that $\hat{\alpha}$ is consistent and find its limit distribution.

(ii) Using data generated from (1), the first order autoregression $Y_t = \hat{\rho}Y_{t-1} + \hat{v}_t$ is fitted. Find the asymptotic behavior of $\hat{\rho}$ as $n \to \infty$.

A2 The exponential trend regression equation

$$Y_t = \alpha\rho^t + \beta\rho^t t + u_t, \quad t = 1, ..., n \tag{2}$$

models the scalar observed time series Y_t in terms of the two trend components ρ^t and $\rho^t t$, where $\rho > 1$ is known, where α and β are unknown coefficients to be estimated, and where u_t is $iid\ N(0, \sigma^2)$ with $\sigma^2 > 0$.

The model (2) is fitted by least squares regression, giving $\hat{\alpha}$ and $\hat{\beta}$.

(i) Show that $\hat{\alpha}$ and $\hat{\beta}$ are consistent and find their limit distribution as $n \to \infty$. Discuss your finding.

(ii) How do your results change when $u_t = \sum_{s=1}^{t} \varepsilon_s$ and $\varepsilon_t \sim iid\,(0, \sigma_\varepsilon^2)$? Indicate any additional assumptions that may be used in your derivations.

(iii) Perform a simulation experiment to examine the finite sample performance of $\hat{\alpha}$ and $\hat{\beta}$ in relation to the asymptotic distribution.

Question B (Explosive Vector Autoregression)

In the bivariate autoregression

$$X_t = AX_{t-1} + u_t, \quad t = 1, ..., n \tag{3}$$

$$A = \begin{bmatrix} \rho & \theta \\ 0 & \rho \end{bmatrix}, \quad \rho > 1, \quad \theta \neq 0, \tag{4}$$

$X_t = (X_{1t}, X_{2t})'$, the error process u_t is $iid\ N(0, \Sigma)$ with $\Sigma > 0$, and the process is initialized at $t = 0$ by $X_0 = 0$. The first equation of (3) is fitted by least squares regression giving

$$X_{1t} = \hat{\rho} X_{1t-1} + \hat{\theta} X_{2t-1} + \hat{u}_{1t}.$$

Find the asymptotic properties of the least squares estimator $\left(\hat{\rho}, \hat{\theta}\right)$ as $n \to \infty$.

Question C (A Scientific Overview Project)

Choose a field of recent econometric research and write a scientific overview paper of that field. The topic can be theory or applied or a combination of the two and it can be in any field of econometrics. The project should be written up as a scientific review paper, covering motivating ideas, explaining the econometric theory, and providing some evalution of the research direction, including its strengths and limitations.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate. This applied project may be related to your Applied Econometrics Paper for the departmental requirement.