Econ. 553a | Peter C. B. Phillips
Yale University | Fall 2008

Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Question A or Question C

> But wait! Has the spirit of econometrics captured your soul? And do you yearn to face a serious research-level problem? If so, then try both Question A and Question B.

Time Allowed: Six weeks
Due Date & Time: Friday 16 January 2009.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu
Any reference material is allowed.

Question A (Estimation under Weak Identification)

The regression model

$$Y_t = \beta_n g(X_t; \pi) + u_t, \quad t = 1, ..., n \tag{1}$$

relates observable time series (Y_t, X_t) and unobservable errors u_t. The unknown regression parameters are $\pi \in \Pi$, a compact set in $\mathbb{R}$, and the loading coefficient $\beta_n = \beta/d_n$, where β is a non-zero constant and where $d_n \to \infty$ as $n \to \infty$. The time series X_t and u_t are independent and strictly stationary with autocovariance sequences $\gamma_x(h) = E(X_t X_{t+h})$ and $\gamma_u(h) = E(u_t u_{t+h})$ which satisfy the summability condition

$$\sum_{h=0}^{\infty} (|\gamma_x(h)| + |\gamma_u(h)|) < \infty.$$

The regression function $g(X_t; \pi)$ and the density, $f(x)$, of X_t are both continuously differentiable with bounded derivatives to the second order.

It is proposed to estimate the systematic component $m_n(x) = \beta_n g(x; \pi)$ of the model (1) by nonparametric regression using the Nadaraya Watson (NW) kernel estimator

$$\hat{m}_n(x) = \sum_{t=1}^{n} Y_t K_h(X_t - x) / \sum_{t=1}^{n} K_h(X_t - x),$$

where $K_h(\cdot) = h^{-1} K\left(\frac{\cdot}{h}\right)$, h is the bandwidth parameter, and the kernel K is continuously differentiable and satisfies $\int_{-\infty}^{\infty} K(z)\,dz = 1$, $\int_{-\infty}^{\infty} zK(z)\,dz = 0$, $\int_{-\infty}^{\infty} z^2 K(z)\,dz = \mu_K^2$, $\int_{-\infty}^{\infty} K(z)^2\,dz = v_K^2 < \infty$.

1. Find the asymptotic properties of $\hat{m}_n(x)$ as $n \to \infty$. Indicate additional assumptions that you use in your derivations.

2. Discuss the optimal bandwidth choice for h based on minimizing a criterion such as asymptotic mean squared error. What happens when $d_n = \sqrt{n}$?

3. Perform a simulation experiment to examine the finite sample performance of $\hat{m}_n(x)$ in the case where $g(X_t; \pi) = (1 + X_t^2)^{\pi}$ for various choices of π and sequences d_n.

Question B (Nonstationary Regression under Weak Identification)

In the regression model

$$Y_t = \beta_n^0 \left(1 + X_t^2\right)^{\pi^0} + u_t, \quad t = 1, ..., n \tag{2}$$

$$\Delta X_t = u_{xt}, \quad X_0 = 0, \tag{3}$$

the error process u_t is stationary and weakly dependent with zero mean and is independent of the scalar regressor variable X_t. It is assumed that partial sums of (u_t, u_{xt}) satisfy the functional law $n^{-1/2} \sum_{t=1}^{[nr]} (u_t, u_{xt}) \Rightarrow (B_u(r), B_x(r))$, where the limit Brownian motions are independent and have variances $\omega_u^2 = \sum_{h=-\infty}^{\infty} \gamma_u(h)$, $\omega_x^2 = \sum_{h=-\infty}^{\infty} \gamma_{u_x}(h)$, respectively, in terms of the autocovariance sequences $\left(\gamma_u(h), \gamma_{u_x}(h)\right)$ of (u_t, u_{xt}).

The unknown regression parameters in (2) are the exponent π, whose true value $\pi^0 > 0$, and the loading coefficient β_n whose true value is $\beta_n^0 = \beta^0/\sqrt{n}$, where β^0 is a non-zero constant.

It is proposed to estimate the parameters $\left(\beta_n^0, \pi^0\right)$ in (2) by nonlinear least squares giving $\left(\hat{\beta}_n, \hat{\pi}\right)$.

Find the asymptotic properties of $\left(\hat{\beta}_n, \hat{\pi}\right)$ as $n \to \infty$.

Question C (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series, cross section or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate.