

Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics Take Home Examination

Answer ONE Question: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Monday 16 January 2005.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu

Question A (Multivariate Explosive Cointegration)

Part 1: In the explosively cointegrated system

$$y_t = Ax_t + u_{0t} \quad (1)$$

$$x_t = \Theta x_{t-1} + u_{xt} \quad (2)$$

$$\Theta = I_K + C, \quad C = \text{diag}(c_1, \dots, c_K), \quad c_i > 0 \quad \forall i, \quad (3)$$

A is an $m \times K$ matrix of ‘cointegrating’ coefficients, x_t is a $K-$ vector of explosively autoregressive time series, which is initialized at $t = 0$ with $x_0 = 0$, and the vector $u_t = (u'_{0t}, u'_{xt})'$ is a sequence of independent, identically distributed $N(0, \Sigma_\varepsilon)$ errors where Σ_ε is positive definite and is partitioned conformably with u_t as follows

$$\Sigma_\varepsilon = \begin{bmatrix} \Sigma_{00} & 0 \\ 0 & \Sigma_{xx} \end{bmatrix}.$$

It is proposed to estimate the matrix A of cointegrating coefficients by least squares regression leading to $\hat{A} = (\sum_{t=1}^n y_t x'_t) (\sum_{t=1}^n x_t x'_t)^{-1}$, with residuals $\hat{u}_t = y_t - \hat{A}x_t$.

1. Suppose $c_i \neq c_j$ for all $i \neq j$.
 - (a) Find a general expression for the asymptotic distribution of \hat{A} as $n \rightarrow \infty$.
 - (b) Consider the special case where $K = 1$. Can you provide a more specific limit result in this case?
2. Suppose $\Theta = (1 + c) I_K$ for some $c > 0$.
 - (a) Find the asymptotic distribution of \hat{A} as $n \rightarrow \infty$.
 - (b) Discuss your results in this case.
3. Perform a small simulation experiment to illustrate some of your findings.

Question B (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series or panel econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation, including graphics of the data and results as necessary. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate.