Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer ONE Question: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 14 January 2005.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu

Question A (Autoregression with Heterogeneous Innovations)

Part 1: The time series X_t is generated by the autoregression

$$X_t = \theta X_{t-1} + g\left(\frac{t}{n}\right) u_t, \qquad t = 1, ..., n; \quad |\theta| < 1,$$

where $X_0 = 0$, u_t is $iid\ N(0,1)$, and where $g(\cdot)$ is a continuous function on the interval $[0,1]$. Both θ and g are unknown.

It is proposed to estimate θ by least squares regression leading to

$$\hat{\theta} = \sum_{t=1}^{n} X_t X_{t-1} / \sum_{t=1}^{n} X_{t-1}^2,$$

with residuals $\hat{u}_t = X_t - \hat{\theta} X_{t-1}$. Let $[a]$ denote the integer part of a.

1. Find $\lim_{n\to\infty} E\left(X_{[nr]}^2\right)$.

2. Find the probability limit of $n^{-1}\sum_{t=1}^{n} X_t^2$ and relate your result to the homogeneous case where g is the constant function $g(r) = \sigma$ for $r \in [0,1]$.

3. Show that the asymptotic distribution of $\hat{\theta}$ is normal and relate the limit distribution to the homogeneous case. Compare the variance of this limit distribution for the functions $g(r) = r^m$ with $m > 0$, $g(r) = \sin^2(\pi r)$, $g(r) = \sin^4(\pi r)$, and $g(r) = \sin^8(\pi r)$. Comment on your results.

4. Find the probability limits of $n^{-1}\sum_{t=1}^{n} \hat{u}_t^2$ and $n^{-1}\sum_{t=1}^{n} \hat{u}_t^4$.

5. Suggest a test of the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ and derive the limit distribution of your test statistic.

Part 2: Perform a simulation experiment to illustrate some of the results you found in Part 1.

Question B (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series econometric methods. Write up your project as a scientific paper, paying attention to the quality of your presentation. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate.