Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer ONE Question: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 9 January 2004.
Electronic Filing: Submit your papers by email to peter.phillips@yale.edu

Question A. (IV Estimation of Simultaneous Equations with Irrelevant Instruments)

Part 1: The bivariate time series $\{Y_t, X_t : t = 1, ..., n\}$ are generated by the simultaneous system

$$Y_t = \beta X_t + u_{0t},$$

$$X_t = \theta + u_{xt}, \quad u_t = C(L)\varepsilon_t, \quad C(L) = \sum_{j=0}^{\infty} c_j L^j, \quad \sum_{j=0}^{\infty} j \|c_j\| < \infty \qquad (1)$$

where $u_t = (u_{0t}, u_{xt})'$, $\varepsilon_t \equiv iid(0, \Sigma_\varepsilon)$ with finite fourth moments, β and θ are unknown parameters, and $\|\cdot\|$ is a matrix norm.. It is proposed to estimate β by instrumental variable (IV) methods using a $q-$ vector of instruments Z_t $(t = 1, ..., n)$ generated by

$$Z_t = D(L)\eta_t, \quad D(L) = \sum_{j=0}^{\infty} d_j L^j, \quad \sum_{j=0}^{\infty} j \|d_j\| < \infty, \qquad (2)$$

where $\eta_t \equiv iid\ N(0, \Sigma_q)$, Σ_q and $\Sigma_Z = \sum_{j=0}^{\infty} d_j \Sigma_q d_j'$ are positive definite matrices, η_t is independent of ε_s for all s and t.

Let $\hat{\beta}_q$ be the IV estimator of β obtained with the instruments Z_t.

1. Show that Z_t satisfies the two orthogonality conditions

$$\frac{1}{n}\sum_{t=1}^{n} Z_t \rightarrow_{a.s.} 0, \quad \frac{1}{n}\sum_{t=1}^{n} Z_t u_t' \rightarrow_{a.s.} 0.$$

2. Find the limit distribution of

$$\begin{bmatrix} \frac{1}{\sqrt{n}}\sum_{t=1}^{n} Z_t \\ \frac{1}{\sqrt{n}}\sum_{t=1}^{n} Z_t \otimes u_t \end{bmatrix}$$

   and prove your result.

3. Find the asymptotic behavior of $\hat{\beta}_q$ as $n \to \infty$.

4. In the special case where $Z_t = \eta_t$ and $\Sigma_q = I_q$, take your result on the asymptotic behavior of $\hat{\beta}_q$ as $n \to \infty$ and show what happens to this limit result when $q \to \infty$. That is, let $q \to \infty$ after you have passed $n \to \infty$.

5. Compare your results with those for the least squares and constant IV (i.e. $Z_t = 1$) estimates of β.

Part 2: Suppose the instruments Z_t are generated by

$$Z_t = \mu t + D(L)\,\eta_t, \quad t = 1, ..., n \tag{3}$$

where μ is a constant vector and $D(L)\,\eta_t$ has the same properties as in (2). Let $\hat{\beta}_q$ be the IV estimator of β in (3) using the instruments Z_t . Analyze the asymptotic behavior of $\hat{\beta}_q$ and compare your results to those you obtained in Part 1.

Part 3: Perform a simulation experiment to illustrate some of the results you found in Parts 1 and 2.

Question B. (Given Empirical Project)

1. Download multi-country macro data files from the Penn World Tables at the Toronto or NBER sites on the internet. Below is the Toronto address. Just follow the instructions for downloading files given at this site. The URL is http://datacentre2.chass.utoronto.ca/pwt/

2. The data set provides annual data for 152 countries. Select the series for real GDP per capita (RGDPC) for the USA and one other country that is of interest to you. Make sure that you have at least 30 observations for each series. With the data you have obtained in this way, perform the following empirical exercise whose object is to estimate the growth rate of real GDP per capita for each country.

   (a) Use the simple trend regression model

   $$y_t = a + bt + u_t, \quad t = 1, ..., n \tag{4}$$

   where y_t is the logarithm of real per capita GDP and u_t is an error process which, in general, is serially correlated. Assume that

   $$u_t = \alpha u_{t-1} + \varepsilon_t \tag{5}$$

   where ε_t is $iid(0, \sigma^2)$.

   (b) Estimate equation (4) under assumption (5) and report your estimates of the growth rate parameter . Try using the following feasible procedures. Report your estimates in a Table so that it is easy for you to consider and comment on your results.

      i. Simple OLS regression of (4);

      ii. Cochrane-Orcutt regression of (4) with $n-1$ transformed observations using an estimate of α in (5) obtained from a preliminary regression that uses OLS regression residuals. Report your estimates of α;

      iii. Cochrane-Orcutt estimation of (4) supplemented with the following equation for the first observation

      $$y_1 = const. + u_1.$$

      That is, use feasible generalized least squares estimation of (4) using n observations.

iv. Simple OLS regression on (4) reformulated in first differences.

(c) Replace (5) with a general $AR(p)$ model for the errors and use the BIC model selection criterion to estimate the order, p, of this autoregression using the residuals from an OLS regression on (4) as data. Use the fitted $AR(p)$ model to transform the data in (4) and then reestimate the parameter . Report the new estimates alongside those obtained by the other methods.

(d) Briefly discuss your results and compare them with those of Canjels and Watson (1997).

Question C. (Your Own Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series econometric methods to economic data investigating some economic issue. Write up your project as a scientific paper, paying attention to the quality of your presentation. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate.

References

Canjels, N. And M. Watson (1997). "Estimating deterministic trend in the presence of serially correlated errors," *Review of Economics and Statistics,* 184-200.

Phillips, P.C.B., and C. C. Lee (1996), "Efficiency Gains from Quasi-Differencing under Nonstationarity" in P.M. Robinson and M. Rosenblatt (Eds.) "*Essays in Memory of E.J. Hannan*". New York: Springer.