Econ. 553a
Yale University

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Fall 2001

Econometrics IV: Time Series Econometrics

Take Home Examination

Answer ONE Question: Any reference material allowed.
Time Allowed: Six weeks
Due Date & Time: Friday 11 January 2001, 12:00 noon.

Question A.(Autoregressive Panel Data Asymptotics)

Consider the following autoregressive model for panel data given by

$$y_{i,t} = \rho y_{i,t-1} + \mu_i + u_{i,t}, \quad \text{for } t = 1, \cdots, T, \quad \text{and } i = 1, \cdots, N, \tag{1}$$

where the μ_i are constant fixed effects, $\rho \in (-1, 1), u_{i,t} \sim iid\ N(0, \sigma_i^2)$ over t and $u_{i,t}$ is independent of $u_{j,s}$ for all $i \neq j$ and for all s, t. It is assumed that

$$\frac{1}{N}\sum_{i=1}^{N} \sigma_i^2 = \sigma^2 + O\left(\frac{1}{N}\right), \text{ as } N \to \infty$$

with $\sigma^2 > 0$ and

$$y_{i,0} \sim \ N(0, \tfrac{\sigma_i^2}{1-\rho^2}) \quad \text{for all } i.$$

The pooled panel least squares estimator of ρ is defined as

$$\hat{\rho}_{pols} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1} - y_{i.-1})(y_{it} - y_{i.})}{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1} - y_{i.-1})^2}, \tag{2}$$

where $y_{i.} = T^{-1}\sum_{t=1}^{T} y_{it}$, and $y_{i.-1} = T^{-1}\sum_{t=1}^{T} y_{it-1}$.

(a) Derive an expression for the asymptotic bias of $\hat{\rho}_{pols}$ as $N \to \infty$ with T fixed, viz.

$$\text{plim}_{N\to\infty}\left(\hat{\rho}_{pols} - \rho\right).$$

(b) Derive the asymptotic distribution of $\hat{\rho}_{pols}$ as both $N, T \to \infty$ and give conditions (including conditions on the passage of N and T to infinity) under which the limit distribution is valid.

(c) Perform a simulation experiment to determine how adequate your asymptotic results are for finite T and N.

Question B. (Empirical Project)

In a study of cross country economic growth, Easterly and Levine (1997) argued that ethnic conflict was an important determinant of poor growth performance in sub-Saharan African economies. Brock and Durlauf (2000) have recently examined the empirical evidence for their findings using Bayesian methods that allow for model uncertainty and heterogeneity uncertainty across countries. Read these two papers and undertake an empirical project that addresses the empirical question of the effect of ethnic conflict on growth.

1. Set up a Bayesian modeling framework like that of Brock and Durlauf (2000) and show how it may be used:

   (a) to find the posterior probability that ethnic conflict affects economic growth;

   (b) to test the proposition that ethnic conflict affects economic growth.

2. Conduct an empirical exercise to perform the analysis that you have suggested in 1(a) and (b). The data set is available on the website http://www.worldbank.org/research/growth/ddeale.htm.

Question C. (Empirical Project)

1. Download multi-country macro data files from the Penn World Tables at the Toronto or NBER sites on the internet. Below is the Toronto address. Just follow the instructions for downloading files given at this site. The URL is http://datacentre2.chass.utoronto.ca/pwt/

2. The data set provides annual data for 152 countries. Select the series for real GDP per capita (RGDPC) for the USA and one other country that is of interest to you. Make sure that you have at least 30 observations for each series. With the data you have obtained in this way, perform the following empirical exercise whose object is to estimate the growth rate of real GDP per capita for each country.

(a) Use the simple trend regression model

$$y_t = a + bt + u_t, \quad t = 1, ..., n \tag{3}$$

where y_t is the logarithm of real per capita GDP and u_t is an error process which, in general, is serially correlated. Assume that

$$u_t = \alpha u_{t-1} + \varepsilon_t \tag{4}$$

where ε_t is $iid(0, \sigma^2)$.

(b) Estimate equation (3) under assumption (4) and report your estimates of the growth rate parameter . Try using the following feasible procedures. Report your estimates in a Table so that it is easy for you to consider and comment on your results.

i. Simple OLS regression of (3);

ii. Cochrane-Orcutt regression of (3) with $n-1$ transformed observations using an estimate of α in (4) obtained from a preliminary regression that uses OLS regression residuals. Report your estimates of α;

iii. Cochrane-Orcutt estimation of (3) supplemented with the following equation for the first observation

$$y_1 = const. + u_1.$$

That is, use feasible generalised least squares estimation of (3) using n observations.

iv. Simple OLS regression on (3) reformulated in first differences.

(c) Replace (4) with a general $AR(p)$ model for the errors and use the BIC model selection criterion to estimate the order, p, of this autoregression using the residuals from an OLS regression on (3) as data. Use the fitted $AR(p)$ model to transform the data in (3) and then reestimate the parameter . Report the new estimates alongside those obtained by the other methods.

(d) Briefly discuss your results and compare them with those of Canjels and Watson (1997).

Question D. (Empirical Project)

Choose your own empirical project. Carry out an empirical application of time series econometric methods to economic data investigating some economic issue. Write up your project as a scientific paper, paying attention to the quality of your presentation. Be sure to provide a full discussion of the methods being used and indicate limitations of the approach you are using wherever you think it is appropriate.

References[1]

Brock W. A. and S. N. Durlauf (2000). "Growth Economics and Reality," Mimeographed, University of Wisconsin.

Canjels, N. And M. Watson (1997). "Estimating deterministric trend in the presence of serially correlated errors," *Review of Economics and Statistics,* 184-200.

Easterly W. and R. Levine (1997), "Africa's Growth Tragedy: Policies and Ethnic Divisions", *Quarterly Journal of Economics*, 1203-1250.

Phillips, P.C.B., and C. C. Lee (1996), "Efficiency Gains from Quasi-Differencing under Nonstationarity" in P.M. Robinson and M. Rosenblatt (Eds.) "*Essays in Memory of E.J. Hannan*". New York: Springer.

[1] Copies of these references are available from Alex in 30 Hillhouse Avenue.