Econ. 553a
Yale University

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Econometrics IV: Time Series Econometrics

Take Home Examination

Answer Questions A and B or just Question C alone: Any reference material allowed. If you answer the empirical question in the optional take home exam, you need only answer one of the questions on this exam.

Time Allowed: Six weeks

Due Date & Time: Friday 12 January 2001, 12:00 noon.

Question A.

Part 1. (Two-Sided BN Decompositions)

Suppose $C(L) = \sum_{j=-\infty}^{\infty} c_j L^j$ and $\sum_{j=-\infty}^{\infty} |j|^{\frac{1}{2}} ||c_j|| < \infty$.

(a) Find the BN and spectral BN decompositions of $C(L)$ and prove their validity.

(b) Suppose $c_j = c_{-j}$ for $j \neq 0$. Does the BN decomposition of $C(L)$ have a special form in this case?

Part 2. (Two-Sided Distributed Lags)

The time series y_t is generated by a two sided distributed lag filter of the form

$$y_t = \sum_{j=-\infty}^{\infty} b_j x_{t-j} = b(L) x_t, \tag{1}$$

where x_t is a stationary time series with Wold representation $x_t = \sum_{j=0}^{\infty} c_j \xi_{t-j}$ where $\xi_t = \text{iid } (0, \sigma_x^2)$ and the coefficients c_j satisfy $\sum_{j=0}^{\infty} j|c_j| < \infty$. Suppose the coefficients in (1) are given by

$$b_j = \begin{cases} \frac{\beta_A \omega_0}{\pi} + \frac{\beta_B}{\pi}(\pi - \omega_0) & j = 0 \\ \frac{\sin j\omega_0}{\pi j}(\beta_A - \beta_B) & j \neq 0 \end{cases}$$

for some finite constants β_A and β_B and for some frequency $\omega_0 \in (0, \pi)$.

(a) Find the transfer function $b(\omega) = \sum_{j=-\infty}^{\infty} b_j e^{ij\omega}$ of the filter $b(L)$ and the spectral density of y_t.

(b) Does the filter $b(L)$ have a valid BN decomposition?

(c) Does the standardized sum $\frac{1}{\sqrt{n}} \sum_{t=1}^{n} y_t$ satisfy a central limit theorem?

Question B.

Part 1

The time series X_t is generated by the model

$$X_t = \alpha + \beta \log\log t + u_t, \quad t = 2, ..., n \tag{2}$$

where α and β are unknown parameters whose least squares regression estimates are denoted by $\widehat{\alpha}$ and $\widehat{\beta}$, respectively. The error u_t in (2) is assumed to be $iid\ (0, \sigma^2)$ with finite fourth moment.

(a) Show that $\widehat{\alpha}$ and $\widehat{\beta}$ are strongly consistent for α and β as $n \to \infty$.

(b) Find the asymptotic distribution of $\widehat{\alpha}$ and $\widehat{\beta}$.

Part 2

Suppose that u_t in (2) is the linear process

$$u_t = \sum_{i=0}^{\infty} c_i \varepsilon_{t-i}, \quad \text{with } \sum_{j=0}^{\infty} j\,|c_j| < \infty, \tag{3}$$

and where ε_t is $iid\ (0, \sigma^2)$ with finite fourth moment. Explain how you would modify your derivations in Part 1 to allow for such an error process in the regression model (2).

Question C. (Empirical Project[1])

In a study of cross country economic growth, Easterly and Levine (1997) argued that ethnic conflict was an important determinant of poor growth performance in sub-Saharan African economies. Brock and Durlauf (2000) have recently examined the empirical evidence for their findings using Bayesian methods that allow for model uncertainty and heterogeneity uncertainty across countries. Read these two papers and undertake an empirical project that addresses the empirical question of the effect of ethnic conflict on growth.

1. Set up a Bayesian modeling framework like that of Brock and Durlauf (2000) and show how it may be used:

   (a) to find the posterior probability that ethnic conflict affects economic growth;

   (b) to test the proposition that ethnic conflict affects economic growth.

2. Conduct an empirical exercise to perform the analysis that you have suggested in 1(a) and (b). The data set is available on the website http://www.worldbank.org/research/growth/ddeale.htm.

References[2]

Brock W. A. and S. N. Durlauf (2000). "Growth Economics and Reality," Mimeographed, University of Wisconsin.

Easterly W. and R. Levine (1997), "Africa's Growth Tragedy: Policies and Ethnic Divisions", *Quarterly Journal of Economics*, 1203-1250.

[1] The empirical project in Question C is a substantial undertaking and could well form the basis of an applied econometrics paper. Students who are more interested in applied econometrics and decide to devote their time to this project should indicate this intent. In that event, the empirical project of Question C will be sufficient for this examination.

[2] Copies of these two references are available from Mary in 30 Hillhouse Avenue.