

## **Econometrics IV: Time Series Econometrics**

### **Course Outline 2013**

This is a one semester version of what was originally a two-course sequence in time series econometrics that included Econ 557b. The course provides an introduction to time series methods in econometrics covering stationary series, aspects of trend behavior, detrending mechanisms and their properties, unit root theory, cointegrated system approaches, realized volatility and quarticity, Wold and BN decompositions, model selection, nonlinear nonstationary models and methods, spatial density asymptotics, and long memory modeling. Both time domain and frequency domain methods are discussed, and Bayesian as well as classical approaches are included. The treatment relies on asymptotic theory for linear processes, martingales and martingale approximations. We overview a large literature and not all topics are treated in the same depth. Theory, computations and some empirical applications are discussed. Most classes are divided into two parts, one dealing with theory and the other with empirics.

No specific text is recommended. The most recent text is Martin, Hurn and Harris (2012), which provides good general coverage at the introductory level in an approach that is oriented towards implementation offering abundant illustrations (uniquely) complete with computer code in Gauss, Matlab and R. Hamilton's (1994)<sup>1</sup> book, Fuller (1996) and Gouriéroux and Monfort (1997) are useful references. Hamilton's coverage is broad and relevant to econometrics, the book is easy to read and it includes much introductory material, but is now dated. Fuller's book provides an accessible statistical treatment of the subject, is a useful revision of an earlier (1976) edition, and was the first text to discuss unit root theory. Gouriéroux and Monfort (1997) is a translation of an excellent French textbook of time series that covers a wide literature from an econometric perspective. Lutkepohl and Kratzig (2004) is a textbook of applied time series econometrics that emphasizes practicalities and covers methods that are popular in empirical economic applications. Brockwell and Davis (1991, with subsequent editions) is a very successful time series text that is commonly used in North American graduate statistics courses. This book is more technical than the above texts and stresses univariate models, but is well expository, covers most of the traditional stationary time series topics and comes with some computer software. Lutkepohl's (1993) book and his newer (2005) text provide excellent coverage and exposition of VAR and Bayesian VAR modelling methods, together with some small scale practical applications to macro data. Hall and Heyde (1980) is a beautifully written classic on martingale limit theory that continues to reward careful reading. Billingsley (1999) is the second edition of a highly influential treatise on weak convergence that first appeared in 1968. Davidson (1994) is a good general reference source on limit theory for econometrics including functional laws, emphasizing mixing and weak dependence. Van de Vaart (1998) is a useful overview of asymptotic methods in statistics, including some empirical process methods. Taniguchi and Kakizawa (2000) give a modern treatment of time series asymptotics from a stochastic process perspective and include some useful special topics like large deviation expansions, saddlepoint approximations and higher order asymptotics. White (2002) provides much useful background and its first edition (1984) was notable for its general treatment of asymptotic covariance matrix estimation.

A take home examination will be given at the end of the course. Students have the option of attempting a solution to the problems in this exam, writing a scientific overview of a modern research area in

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<sup>1</sup> See Section 0 in the Reading Guide below for general references.

econometrics, or doing an applied econometrics paper on a topic of their choice. The empirical paper may be used for the applied econometrics paper requirement. Past take home exams over many years are all available on the web and some solution sets are available.

The following is a general outline of how we proceed through the course material. Some of the material is taught in section based on earlier years' lectures to help us cover more material in the course. We adjust lecture content according to the rate of progress, importance of the material, and relevance to applied work. Some empirical applications will be discussed.

<b>Week</b>	<b>Content</b>
1 & 2	Ideas and approaches to time series. Primary concerns and methods of inference: Classical, Bayesian and prequential approaches. Role of unit roots and cointegration in econometric modeling. Brownian motion, the Karhunen-Loève representation, and some of its recent applications.
3 & 4	Heuristic ideas and implications for inference and modelling. Simple parametric models, including VARs, and Cointegrated systems. Optimal estimation and IV approaches. Some preliminary asymptotics. Information criteria and model selection asymptotics. Trend Elimination.
5 & 6	Ergodicity and notions of weak dependence. Conditional expectations and Hilbert projections. The Wold decomposition and forecasting. Grenander Rosenblatt theory.
7 & 8	The Phillips-Solo device & shortcuts to time series asymptotics. Strong laws and CLT's for time series. Unit root limit theory, asymptotic degeneracy, realized volatility, quadratic variation, and quarticity. Optimal estimation of cointegrated systems.
9 & 10	Martingales and time series applications of the martingale convergence theorem. Explosive and mildly explosive time series. Bubbles, crashes and applications in finance.
11	Nonlinear nonstationary models, spatial density, estimation of local time. Applications to market intervention.
12 & 13	Frequency domain approaches and spectral regression. Spectral density and long run variance estimation. Long memory models and econometric methods. More on unit roots and cointegration.
<b>December - January</b>	<b><i>Take Home examination paper, overview paper, or applied econometrics paper</i></b>

## Reading Guide

Time series is a vast subject. The following list covers only that part of the subject that relates most closely to econometric research. The list is subdivided into topics that are relevant to material we intend to discuss, if only briefly in some cases, during the course.

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<sup>2</sup> Asterisked references are more important to the course.

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## 1. Ideas and Approaches

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