

92.3.5. *Efficiency of Maximum Likelihood*, proposed by Peter C.B. Phillips. In the linear model

$$y_t = bx_t + u_t, \quad (t = 1, \dots, n) \quad (1)$$

the parameter has true value  $b_0 \neq 0$  and  $u_t \equiv$  i.i.d.  $N(0, b_0^2)$ . The  $x_t$  are non-random and  $n^{-1} \sum_1^n x_t^2 \rightarrow m_x > 0$  as  $n \rightarrow \infty$ .

- (i) Derive the asymptotic properties of the maximum likelihood estimator  $\tilde{b}$  of  $b_0$  in (1).
- (ii) Compare the limit distribution of  $\tilde{b}$  to that of the OLS estimator  $\hat{b}$  of  $b_0$  in (1). Is OLS asymptotically efficient?