

92.1.5. *Limit Theory in Cointegrated Vector Autoregressions*, proposed by Peter C.B. Phillips and Hiro Y. Toda. Consider the trivariate system

$$y_t = Ay_{t-1} + u_t, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

where u_t is i.i.d. $(0, \Sigma)$ and $\Sigma = (\sigma_{ij})$. Suppose $A = (a_{ij})$ is estimated by an unrestricted vector autoregression with one lag leading to $\hat{A} = (\hat{a}_{ij})$.

- (a) Find the joint asymptotic distribution of $(\hat{a}_{12}, \hat{a}_{13})$.
- (b) Find the limit distribution of the Wald test of the noncausality hypothesis.

$$\mathcal{H}: a_{12} = a_{13} = 0.$$

- (c) Discuss your results.