90.2.5. *Optimal Structural Estimation of Triangular Systems: I. The Stationary Case*, proposed by P.C.B. Phillips. Consider the structural model

$$y_{1t} = \beta y_{2t} + u_{1t} \tag{1}$$

$$y_{2t} = \gamma' x_t + u_{2t} \tag{2}$$

where $t = 1, \ldots, n$, $u_t = (u_{1t}, u_{2t})'$ is iid $N(0,\Sigma)$ with covariance matrix $\Sigma = \sigma^2 \Sigma_0$ and

$$\Sigma_0 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

is a known matrix. The vector x_t in (2) is exogenous, uniformly bounded and $n^{-1}\Sigma_1^n x_t x_t'$ converges to the positive definite matrix M as $n \to \infty$.

An econometrician (A) wishes to obtain asymptotically efficient estimates of the parameter β in equation (1). Since (2) is a reduced form equation, he argues that two stage least squares (2SLS) is optimal. A colleague (B) points out that Σ_0 is known and the model can therefore be reduced to a truly triangular system (i.e. with triangular structural coefficient matrix and diagonal error covariance matrix) by subtracting equation (2) from equation (1) leading to a revised model whose first equation is now:

$$y_{1t} = (1 + \beta)y_{2t} - \gamma' x_t + v_{1t}. \qquad (1)'$$

Econometrician B argues that $v_{1t} = u_{1t} - u_{2t}$ is independent of u_{2t}, (2) is still a reduced form and therefore ordinary least squares (OLS) on (1)′ are asymptotically efficient.

Find the limit distributions of estimates of β obtained by 2SLS on (1), OLS on (1)′ and full system maximum likelihood. Which econometrician is right, or are both of them wrong?