90.2.6 *Joint Estimation of Equilibrium Coefficients and Short-Run Dynamics*, proposed by P.C.B. Phillips. Consider the model

$$y_t = x_t'\beta + z_t'\gamma + u_{1t},$$

$$z_t = u_{2t},$$

$$x_t = x_{t-1} + u_{3t}, \tag{1}$$

where $t = 1, \ldots, n$, $x_0 = 0$ and $u_t' = (u_{1t}, u_{2t}, u_{3t})$ is strictly stationary and ergodic with zero mean and covariance matrix Σ. The vector z_t in (1) is strictly exogenous (i.e. $E(z_t u_{1s}) = 0$ for all t and s), and u_{1t} is a martingale difference sequence with respect to the filtration $\mathfrak{F}_{t-1} = \sigma(u_{1t-1}, u_{1t-2}, \ldots; (u_{2s}, u_{3s})_{-\infty}^{\infty})$, i.e. $E(u_{1t}|\mathfrak{F}_{t-1}) = 0$, and u_t satisfies the invariance principle $n^{-1/2}\Sigma_1^{[nr]} u_t \to_d B(r) \equiv \mathrm{BM}(\Omega)$ where

$$\Omega = \begin{bmatrix} \omega_{11} & 0 & 0 \\ 0 & \Omega_{22} & \Omega_{23} \\ 0 & \Omega_{23}' & \Omega_{33} \end{bmatrix}. \tag{2}$$

Let $\hat{\beta}$ and $\hat{\gamma}$ be the OLS estimators of β and γ in (1). Find the limit distributions and joint limit distributions of these estimators. What do you conclude from these results about the nature of the dependence between estimators of the long-run equilibrium coefficients β and the short-run or stationary dynamic coefficients γ?