

91.3.3. *Testing for Stationarity in the Components Representation of a Time Series*, proposed by D. Kwiatkowski, P.C.B. Phillips, and P. Schmidt. Let the time series  $(y_t)_{t=1}^n$  have the following components representation

$$y_t = \gamma_0 + \gamma_1 t + r_t + u_t; \quad u_t = \text{i.i.d. } N(0, \sigma_u^2)$$

where

$$r_t = r_{t-1} + v_t; \quad v_t = \text{i.i.d. } N(0, \sigma_v^2), \quad r_0 = 0$$

and  $u_t$  is independent of  $v_s$  for all  $s$  and  $t$ .

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- (a) Set up the likelihood function for this model.
- (b) Derive an LM statistic for testing the hypothesis

$$H_0: \sigma_v^2 = 0.$$

- (c) Find the limit distribution under  $H_0$  of the LM statistic derived in (b).
- (d) How do your results depend on the normality assumption?