88.1.3 *Structural Estimation under Partial Identification*, proposed by P.C.B. Phillips. Consider the following single equation of a simultaneous system:

$$y_1 = Y_2\beta_2 + Y_3\beta_3 + X_1\gamma + u \tag{1}$$

with reduced form

$$T[\overset{1}{y_1},\ \overset{n_2}{Y_2},\ \overset{n_3}{Y_3}] = [\overset{K_1}{X_1},\ \overset{K_2}{X_2}] \begin{bmatrix} \overset{1}{\Pi_{11}} & \overset{n_2}{\Pi_{12}} & \overset{n_3}{\Pi_{13}} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \end{bmatrix} + [\overset{1}{v_1},\ \overset{n_2}{V_2},\ \overset{n_3}{V_3}] \tag{2}$$

or

$$Y = X\Pi + V. \tag{3}$$

Suppose $\Pi_{23} = 0$, rank $(\Pi_{22}) = n_2$, $\beta_2 = 0$, and $K_2 \geq n_2 + n_3$. Assume that standardizing transformations have been carried out, so that the rows of

$[y_1, Y_2, Y_3]$ are i.i.d. $N(0, I_n)$, where $n = 1 + n_2 + n_3$. Let $(\hat{\beta}_2', \hat{\beta}_3')$ denote the two-stage least squares (2SLS) estimator of (β_2', β_3').

(i) Find the distribution of $\hat{\beta}_3$. What happens to this distribution as $T \uparrow \infty$?

(ii) Assuming that $X'X/T$ tends to a positive definite matrix as $T \uparrow \infty$, how would you analyze the asymptotic distribution of $\hat{\beta}_2$? How do your results accord with conventional asymptotic theory?