86.2.5. *The Distribution of LIML in the Leading Case*—Solution, proposed by Peter C.B. Phillips. Let $X = [\overset{1}{X_1}, \overset{n}{X_2'}]' \equiv N(0, I_m)$, where $m = n + 1$ and where we use the symbol "$\equiv$" to signify equality in distribution. The vector $h = X(X'X)^{-1/2}$ is distributed uniformly on the unit sphere in $\mathbb{R}^m$ and we may therefore write the LIML estimator in the form:

$$\tilde{\beta} \equiv -h_2/h_1 = -X_2/X_1 \equiv N(0, I_n)/N(0,1) \equiv \text{multivariate Cauchy}.$$

The final $\equiv$ follows by elementary integration. Thus, setting $r = -X_2/X_1$ and $s = X_1^2$, we have

$$\begin{aligned}\text{pdf}(r) &= (2\pi)^{-m/2} \int_0^\infty e^{-(1+r'r)s/2} s^{m/2-1} ds, \\ &= \Gamma(m/2)\pi^{-m/2}(1 + r'r)^{-m/2}\end{aligned}$$

as required.