Use the Kalman filter technique to obtain the *analytic* expression for the exact log-likelihood function of $(x_1, x_2, \ldots, x_t, \ldots, x_T)'$.

85.1.4 *A Nonnormal Limiting Distribution*, proposed by Peter C. B. Phillips. In the structural equation

$$y_1 = y_2\beta + Z_1\gamma + u,$$

$\mathbf{y}_1$ and $\mathbf{y}_2$ are $T \times 1$ observation vectors, Z_1 is a $T \times K_1$ matrix of observations of exogenous variables and u is a $T \times 1$ vector of random disturbances. The reduced form equations for $(\mathbf{y}_1, \mathbf{y}_2)$ are

$$(\mathbf{y}_1, \mathbf{y}_2) = Z_1\Pi_1 + (v_1, v_2) = Z_1\Pi_1 + V$$

Note that β is not identifiable with this reduced form. If $\mathrm{H} = [\mathrm{Z}_1 \ \vdots \ \mathrm{Z}^*]$ is a matrix of $K_1 + 1$ instruments of full rank, the instrumental variable estimator of β is given by

$$\begin{aligned} b &= [y_2'(P_H - P_{Z_1})y_2]^{-1}[y_2'(P_H - P_{Z_1})y_1] \\ &= (y_2'cc'y_2)^{-1}(y_2'cc'y_1) \\ &= X_1/X_2, \end{aligned}$$

where $P_A = A(A'A)^{-1}A'$, c is the (orthonormalized) latent vector that spans the range space of $P_H - P_{Z_1}$ and $X_1 = c'y_1$, $X_2 = c'y_2$.

1. If (X_1, X_2) has a uniform distribution over the rectangle $[-a, a] \times [-a, a]$ show that the probability density of b is

   $$\mathrm{pdf}(b) = \begin{cases} \frac{1}{4} & -1 \leqslant b \leqslant 1 \\ 1/4b^2 & |b| > 1. \end{cases}$$

2. If the rows of V, that is (v_{1t}, v_{2t}) have independent uniform distributions over the rectangle $[-a, a] \times [-a, a]$ show that the limiting distribution of b is Cauchy.
3. Does the result in Problem 2 hold under more general conditions?