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TRENDING MULTIPLE TIME SERIES

Editor's Introduction

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1. INTRODUCTION

One of the more obvious empirical characteristics of macroeconomic time series is their tendency to grow, or trend, over time. Dealing with this trend-nonstationarity in models of multiple time series has been a major agenda of econometric research for much of the last decade and has produced an enormous literature. Equally, the goal of developing a general asymptotic theory of inference for stochastic processes has been a long-standing concern of probabilists and statisticians. Finally, understanding and modeling trend processes and cyclical activity lie at the nerve center of much of modern macroeconomics. As a consequence, research on nonstationary time series has brought statisticians, econometricians, and macroeconomists close together in productive ways that simply could not have been anticipated 10 years ago.

The focus of this symposium issue of *Econometric Theory* is inference from multiple time series data with trends, and the symposium brings together researchers with these diverse interests. The papers included in the issue were, with two exceptions, presented at a conference called "Trending Multiple Time Series," held at Yale University in the fall of 1993 under the financial sponsorship of the National Science Foundation. All of the papers were written by conference participants. The conference was the fourth in a series of small conferences at Yale on the general theme of "Applications of Functional Limit Theory to Econometrics and Statistics." During the last 10 years, functional central limit theory has become part of the standard operating toolkit of time series econometrics; it has broken new ground in the way in which we do time series asymptotics; it has led to a substantively new understanding of regressions among time series with stochastic trend components; and in doing so it has helped macroeconomists interpret relationships between the levels of economic time series, as distinct from detrended and filtered versions of these series. From this perspective, the present symposium of papers is very closely related to the overall theme of the Yale conference series; and, in concert with one of the original intentions

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of the series, the symposium presents the research of diverse conference participants whose primary interests lie in the fields of probability and statistics, econometrics, and macroeconomics.

The conference "Trending Multiple Time Series" was held over a 2-day period in October 1993. The program is printed at the end of this series of papers, together with some conference photographs. Twenty-two visitors from outside of Yale attended the conference, four of them from overseas. Thanks must go to authors, to conference participants, and to external referees for their combined efforts in making possible this publication of the conference proceedings.

2. OVERVIEW OF THIS SYMPOSIUM ISSUE

Most of the papers assembled in this issue are the response to an invitation that was made early in 1993 to contribute to the subject of trending multiple time series. No directives were given regarding the balance of theoretical and empirical research, and no preferences were expressed regarding topics of special interest in organizing the conference, although there was, of course, a good deal of prior information about the research interests of many of the investigators. The resulting collage of papers therefore represents concerns and interests of people working in the field of nonstationary time series at the present time. The following specific topics are covered in this collection of papers:

asymptotic theory and time series applications (Jeganathan; Saikkonen),

robust nonstationary regression (Phillips),

testing for cointegration (Choi and Ahn; Horvath and Watson; Toda),

regression with mixtures of integrated processes (Chang and Phillips; Kitamura and Phillips),

models with nearly integrated regressors (Cavanagh, Elliott, and Stock), and

unit root tests with covariates (Hansen).

The discussion that follows gives an overview of each of these contributions.

Jeganathan provides a general approach to asymptotic theory for time series that allows for unit roots, cointegration, and stationary processes. This approach utilizes LeCam's (1960, 1986) framework of local asymptotically quadratic (LAQ) approximations to the likelihood ratio. Jeganathan shows how these LAQ approximations in the parameters specialize to the cases of local asymptotic normality (LAN) and local asymptotic mixed normality (LAMN) for models of stationary, cointegrated, and certain Gaussian explosive processes. But, when there are unit roots in the time series model, the approximation to the likelihood has the form of a local asymptotic Brownian functional (LABF). A related result was given in Phillips (1989), but

Jeganathan's classification has deeper implications for inference, because the optimality theory of estimation that applies in LAN and LAMN models does not apply directly to situations where LABF is relevant. Jeganathan's analysis reveals that it is the nonancillary quadratic component in the LABF that stands in the way of the usual efficiency results. Although the LABF applies only at isolated points in the parameter spaces, these isolated points (on the unit circle) are precisely those that are of statistical interest. Indeed, it is the variable random information (in the terminology used in Phillips, 1989) when there is a unit root that causes problems, such as the inconsistency of bootstrap estimates (Basawa, Mallik, McCormick, Reeves, and Taylor, 1991), that have recently attracted interest. The scope of models considered in Jeganathan's study is large and includes some nonlinear time series models and categorical time series models as well as linear models with unit roots and cointegration. In addition to maximum likelihood estimators and *M*-estimators, Jeganathan also considers adaptive time series estimation and outlines some of the essential ingredients of this approach in a simple cointegrated regression context. Jeganathan's paper is a long one. It represents a vast drawing together of ideas and methods from general asymptotic theory to time series models, showing the range of the general theory and the significance of the exceptions. Jeganathan's exposition is accessible to most technically able time series econometricians, and a careful study of his paper is rewarding as much for the understanding that it imparts as for the ideas it provides for subsequent research. Jeganathan's paper was originally written in 1988 and has circulated widely in the econometrics community since then. We are fortunate to be able to include it in this issue.

Saikkonen discusses a variety of difficulties that arise in the development of an asymptotic theory of nonlinear maximum likelihood estimation in integrated and cointegrated systems. Two central difficulties are that when integrated and cointegrated processes appear in a model the rates of consistency are different in different directions, and the Fisher information is random. In linear models, rotations of the regressor space are sufficient to deal with this issue (Park and Phillips, 1988; Phillips, 1989), and the implications are now rather well-known (e.g., Sims, Stock, and Watson, 1990; Toda and Phillips, 1993; Watson, 1995). Nonlinear models present more severe problems, and Saikkonen sets out to characterize these. He shows some sufficient conditions, including a stochastic equicontinuity condition on the Fisher information matrix, which can be used to get results and which illustrate their application to some simple time series models with nonlinearities in the parameters.

Phillips develops a robust statistical approach to estimation and inference in cointegrating regression. Many applications of cointegration are to financial data (like exchange rates, common stock prices, and dividends returns) for which outlier activity is very common. In such cases, we may expect methods that are based on least squares or Gaussian maximum likelihood to be more affected by the presence of outliers in the data than robust methods like M-estimates and least absolute deviations (LADs). Phillips gives robust estimation methods for cointegrating regressions that involve semi-parametric modifications to the usual M-estimation and LAD estimation procedures. The modifications are designed to deal with serial correlation in the data of a general form and endogeneities in the regressors. The new estimators are called fully modified M (FM-M) and fully modified LAD (FM-LAD) estimators. In developing an asymptotic theory for these robust FM estimators, the paper introduces the concept of a generalized function of a random variable. The concept is useful because the objective function in LAD and some M-estimation problems is not smooth; therefore, conventional Taylor series expansions cannot be used to do the asymptotics. The paper's approach is to treat such objective functions as generalized functions and use generalized Taylor series expansions to extract the asymptotics. This involves a new strong law of large numbers and some weak convergence theory for partial sums of generalized functions of random variables. Experiments with simulated data show that when the data generating process has heavy tailed error distributions, there is much to be gained from the use of the robust FM estimation procedures. An empirical application to exchange rate data confirms that outlier activity in the data can lead to major differences, not only in estimation but also in inference, between least squares based and robust estimation methods in nonstationary regressions.

Choi and Ahn introduce LM tests for the null of cointegration in a system of equations. The tests are constructed using the residuals from an optimal cointegrating regression, such as the fully modified regression estimator of Phillips and Hansen (1990) or the canonical cointegrating regression estimator of Park (1992). Use of these residuals makes the LM tests asymptotically free of nuisance parameters under the hypothesis of full cointegration in the system as specified in the null model. These tests may therefore be regarded as specification tests of a cointegrated model. Choi and Ahn give the relevant asymptotics for their test under both null and alternative hypotheses. Simulations show that the tests can suffer from finite sample size distortions like those of unit root tests (Schwert, 1989), although these distortions are reduced when a direct kernel estimate of the long-run conditional covariance matrix of the cointegrating regression equation error is used in setting up the LM test. As in the related univariate KPSS test for stationarity (see Kwiatkowski, Phillips, Schmidt, and Shin, 1992), finite sample and asymptotic power do depend on the bandwidth choice. In particular, power divergence rates are related to the bandwidth expansion rate as a function of the sample size.

Horvath and Watson consider tests for cointegration when some of the cointegrating vectors are prespecified. Such strong forms of cointegration arise when economic theory delivers the cointegrating relation directly by characterizing the form of ratios, differences, or "spreads" between variables. The paper develops an asymptotic theory for Wald tests of strong cointegration using a Gaussian vector error correction model, and these tests are extended by permitting some elements of the cointegrating vectors to be unknown. An empirical application testing the stability of the forward rate–spot rate premium in the foreign exchange rate market illustrates the utility of this approach to cointegration tests.

Toda studies the finite sample performance of the likelihood ratio cointegrating rank tests that were proposed by Johansen (1988, 1991). The model is transformed into canonical form, so that the key parameters affecting the performance of the tests are isolated and then systematically varied in a comprehensive simulation study. The findings reveal that the performance of the test is very sensitive to the stationary roots in the system and is heavily dependent on the correlation between the innovations driving the stationary and nonstationary components of the system. Two general conclusions to emerge from this study are (a) at least 300 observations are needed to ensure reasonably good performance of the tests uniformly over these nuisance parameters and (b) the sequential test procedure lacks discriminatory power between roots at unity and roots close to unity, just like the analogous augmented Dickey–Fuller (ADF) univariate unit root test procedure. In particular, 100 observations do not seem to be sufficient to detect the true cointegrating rank when there is a stationary root in the system that is 0.8 or greater.

Chang and Phillips develop a statistical theory for regressions that contain mixtures of integrated processes. In practical applications, investigators are often unsure whether unit roots are present, and whether there is cointegration in the data. There is also uncertainty about the rank of the cointegration space and sometimes even the order of integration of the time series. As a consequence, empirical methods that are now in common use involve pretesting the data to resolve these issues prior to setting up a form of the model for use in estimation, inference, or prediction. Phillips (1995) proposed an alternative procedure that allows for estimation and inference in cases of such uncertainty without pretesting. The procedure involves unrestricted estimation by fully modified least squares (FM-OLS). Chang and Phillips provide an extension of this approach, called residual-based fully modified least squares (RBFM-OLS), that allows for the presence of I(2) variables, as well as possibly I(1) and I(0) data and unknown degrees of cointegration. An asymptotic theory is developed, and the estimator is shown to provide asymptotically efficient estimates of the cointegration space even though the dimension of the space is unknown. The asymptotic treatment of RBFM-OLS when I(0), I(1), and I(2) variates are present involves a substantial degree of complexity, although the estimator itself has a simple form and is easy to calculate. As part of its development, the paper provides a large number of limit results that are likely to be useful in other time series applications.

Kitamura and Phillips study another extension of the FM-OLS estimation procedure, one that allows for instrumental variables. When there are stationary or cointegrated regressors in a regression model with integrated regressors, the associated I(0) variables need to be instrumented in estimation in order to cope with their endogeneity and avoid simultaneous equations bias. In fully stationary systems, methods such as generalized instrumental variables estimation (GIVE) and generalized method of moments (GMM) have been developed to provide efficient versions of instrumental variables (IV) estimation. In related work, Kitamura and Phillips (1992) devised extensions of these procedures to cope with nonstationary components in the regressors as well. The new methods are called fully modified GIVE (FM-GIVE) and fully modified GMM (FM-GMM) estimators. The paper by Kitamura and Phillips provides an overview of these techniques and investigates their finite sample performance in simulations that allow for regressors with both stationary and nonstationary components, but where there is no knowledge in estimation concerning the directions of stationarity and nonstationarity. The findings reveal that the FM-GIVE estimator generally outperforms the FM-GMM and FM-IV procedures with regard to the stationary coefficients, whereas all of the estimators have similar performance with regard to the nonstationary coefficients. The results seem especially encouraging for the FM-GIVE estimator, which gives asymptotically efficient estimates of both the nonstationary and the stationary coefficients, without prior knowledge of which is which.

Cavanagh, Elliott, and Stock examine a bivariate regression model where the focus of attention is testing the causal (or predictive) effect of the regressor, when the regressor may have a large autoregressive root. The paper investigates the size distortions in conventional t-tests and two-stage tests wherein the critical values of the tests are selected by a preliminary test procedure like the ADF unit root test. Using bounds procedures, like those of Scheffe and Bonferroni, the paper constructs tests and confidence intervals that are asymptotically valid in the sense that size is controlled for autoregressive parameters in the locality of unity. These test procedures are conservative in the sense that the nominal asymptotic size is an upper bound. Simulations reported in the paper show that the procedures are effective in controlling size in finite samples and, for many parameter values, do not suffer substantial power loss.

Hansen points out that the conventional practice of implementing univariate unit root tests can lead to a power loss when there are related stationary covariates that can be included in the regression equation. The paper gives an asymptotic theory for a covariate-augmented version of the usual ADF test (called the CADF test) and shows that the limit theory is a mixture of unit root and standard normal variates, in which the mixture weighting depends on the correlation between the equation error and the stationary regression covariates. Local asymptotic power functions of the CADF test are computed, and these indicate substantial power gains from the use of covariates. Simulations confirm these gains in finite samples, at least when the correct covariates are included. In practical applications of this procedure, modeling decisions need to be made concerning the choice of covariates and the form (levels or differences) in which they are included. The CADF test is applied to the Nelson–Plosser data and supports the hypothesis that real per capita GNP in the United States is stationary but highly persistent and that the industrial production series has a unit root.

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