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A Reexamination of the Consumption Function Using Frequency Domain Regressions

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Abstract: This paper reexamines the permanent income hypothesis (PIH) in the frequency domain. In contrast to some time domain tests, our frequency domain approach provides an explicit and natural test of *both* the permanent *and* transitory implications of the PIH for jointly nonstationary consumption and income data. Using a simple theoretical model, we demonstrate that the PIH implies the marginal propensity to consume (MPC) out of zero frequency income is unity. The PIH *also* implies that the theoretical MPC out of transitory (or high frequency) income is smaller than the long-run MPC. These theoretical restrictions are natural implications of Friedman's hypothesis that agents consume out of permanent or low frequency income and (dis)save out of transitory or high frequency income.

We test this full set of restrictions directly using spectral regression techniques. Under our set of assumptions, the derived disposable income process is shown to have a unit root and to be cointegrated with consumption. We therefore employ a systems spectral regression procedure that accommodates stochastic trends in the consumption and income series as well as the joint dependence in these series. In view of the relatively recent development of these systems spectral estimators, we also conduct Monte Carlo simulations across both low and high frequencies to assess properties of the estimator relative to established single equation techniques. New empirical estimates of the consumption function and tests of the PIH based on systems spectral regression methods are reported for U.S. aggregate consumption and income data over the period 1948–1993. The empirical results provide some evidence for the theoretical implications of the PIH.

Key Words: Spectral Regression, Co-integrated System, Permanent Income Hypothesis.

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I Introduction

In numerous economic applications, it may be more natural to test a proposed theory in the frequency domain than in the time domain. One example is the

permanent income hypothesis (PIH), which suggests that agents consume out of permanent (or low frequency) income and save or dissave out of transitory (or high frequency) income. As a specific illustration of the PIH, we use a simple theoretical model and particular parameterizations of the wage process to demonstrate that the derived disposable income process has a unit root and is cointegrated with consumption. The PIH implies a specific cointegrating relation; the marginal propensity to consume (MPC) out of zero frequency disposable income is unity. However, a simple test that the cointegrating vector between consumption and income is $(1, -1)$ is not sufficient to accept the PIH. The PIH *also* implies that the theoretical MPC out of transitory (or high frequency) income is smaller than the long-run MPC for many parameterizations of the wage process. Thus, the PIH implies specific restrictions across *both* low *and* high frequency coherence in the consumption and income data.

There are numerous papers that study the PIH in the time domain. Stock (1988) argues that, with cointegrated consumption and disposable income series, any difference between the average and marginal propensities to consume is attributable to small sample bias rather than an errors-in-variables problem as previously suggested. His argument concentrates only on the zero frequency implications of the PIH. Quah (1990) attempts to reconcile the Deaton Paradox that observed consumption is too smooth to be generated by the PIH if labor income is difference stationary. When agents distinguish between permanent *and* transitory movements in their labor income, then the PIH predicts that consumption volatility is a "mixture" of the highly volatile prediction under difference stationarity and the smooth prediction under trend stationarity, despite the fact that the econometrician would observe labor income to be difference stationary. In order to capture high frequency restrictions implied by the PIH in the time domain, Campbell (1987) imposes a set of restrictions across a VAR which includes savings and the change in labor income. By focusing on *both* low and high frequency restrictions, we view our paper as a complementary study to Campbell in the frequency domain. One advantage of our approach, however, is that a rejection of the PIH due to "excess sensitivity of consumption" as in Flavin (1981) is easily interpreted in the frequency domain.

A study of the PIH in the frequency domain is not new. In this paper we build on the work of Engle (1974), who first applied band spectral regression techniques to test the PIH.[2] Engle conducted a simple test of the hypothesis that permanent and transitory components have the same marginal propensity to consume. The statistical analysis was based on single-equation band spectral regression. This paper extends Engle's (1974) work to take into account the joint dependence in the consumption and income series, as well as the presence of stochastic trends. We base our empirical analysis on the Engle and Granger (1987) theory of co-integration and Phillips' (1991) spectral regression procedure

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[2]   There are numerous other studies that employ band spectral regression techniques to test a given hypothesis. See, for instance, B. Raj and P. Siklos (1988).

for co-integrated regressions. Unlike single equation spectral regression, which does not properly deal with stochastic trends, the system procedure employed in this paper yields estimates of the long-run MPC which are efficient and asymptotically median unbiased. Hypothesis testing is conducted using standard asymptotic chi-squared tests.

The paper is organized as follows. Section II illustrates the utility of our frequency domain approach by providing a simple representative agent's decision problem, and shows that the MPC is a decreasing function of the frequency for many parameterizations of the wage process; a restriction we subsequently test. Section III outlines the econometric methodology we employ to estimate the MPC. Section IV describes Monte Carlo experiments designed to evaluate the systems spectral estimator across low and high frequencies relative to single equation band spectral regression procedures. Section V examines the time series and frequency domain properties of the consumption and income data used in our empirical study. We provide estimates of the MPC from systems band spectral techniques. Tests of the constancy of the MPC across frequency bands are also performed. Section VI gives concluding remarks.

II Consumption Theory

To illustrate the properties of the PIH in the frequency domain, we adopt a simple version of a representative agent's problem outlined in Sargent (1987). Our agent chooses a plan for consumption that maximizes the present discounted expected value of a quadratic utility function

$$U(C_t) = -\tfrac{1}{2}(b - C_t)^2 + C_t \varepsilon_t^c$$

where b is the bliss level of consumption and ε_t^c denotes stochastic preference shocks. We assume that ε_t^c is a serially uncorrelated random process with mean zero which is uncorrelated with the labor income process w_t (that is, $E[\varepsilon_t^c w_t] = 0$, $\forall t, s$). The agent knows present and past values of ε_t^c, but not future values at the time of choice.

The problem is to maximize

$$E_t\left[\sum_{j=0}^{\infty} \rho^j U(C_{t+j})\right] \tag{1}$$

subject to the sequence of budget constraints

$$A_{t+1} = R[A_t + w_t - C_t] \tag{2}$$

and the boundary conditions

$$A_0 \text{ given and } \lim_{j\to\infty} R^{-j}A_{t+j} = 0 \quad a.s.$$

where ρ is the discount factor $(0 < \rho < 1)$, $R = (1 + r)$ is the gross return on wealth between t and $t + 1$, A_t is assets or indebtedness, and w_t is an exogenously given process for disposable labor income. Under the assumption $\rho = R^{-1}$, the first order conditions are given by

$$E_t \rho R U'(C_{t+1}) = U'(C_t) \quad or \quad E_t C_{t+1} = C_t - \varepsilon_t^c \tag{3}$$

Recursions on (3) can be substituted into the consolidated budget constraint to yield

$$C_t = (1 - R^{-1})\left[A_t + \sum_{j=0}^{\infty} R^{-j} E_t w_{t+j} \right] + \varepsilon_t^c \tag{4}$$

which states that consumption is equal to permanent income plus a transitory preference shock. Lastly, we note that total disposable income can be expressed as $Y_t = (1 - R^{-1})A_t + w_t$.

For simplicity, we assume that disposable labor income is governed by the $IMA(1,1)$ process $(1 - L)w_t = (1 - \gamma L)\varepsilon_t^w$, where ε_t^w is serially uncorrelated. This process implies that consumption and income are co-integrated since[3]

$$C_t - Y_t = -\gamma R^{-1}\varepsilon_t^w + \varepsilon_t^c \tag{5}$$

The disposable labor income process and the asset transition equation implies that total disposable income can be represented by the following integrated stochastic process

$$(1 - L)Y_t = (1 - \gamma R^{-1}L)\varepsilon_t^w - (R - 1)L\varepsilon_t^c \tag{6}$$

Equations (5) and (6) yield the PIH consumption function:

$$C_t = \left(\frac{1 - R^{-1}}{1 - \gamma R^{-1}L} \right) Y_t + \left(\frac{1 - \gamma L}{1 - \gamma R^{-1}L} \right) \varepsilon_t^c \tag{7}$$

We now discuss the frequency domain version of (7). Let (C_t, Y_t) be discrete time series with spectral representations

$$C_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ_C(\lambda) \quad and \quad Y_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ_Y(\lambda) \tag{8}$$

where (Z_C, Z_Y) are random processes with orthogonal increments (and possibly infinite variances at $\lambda = 0$ to accommodate the $I(1)$ nature of C_t and Y_t). Let $Z_u(\lambda)$ be a random process with orthogonal increments and finite variance $E[Z_u(\lambda)Z_u(\lambda)^*] = F_u(\lambda)$. We can represent (7) in the frequency domain as

$$dZ_C(\lambda) = \beta(\lambda)dZ_Y(\lambda) + dZ_u(\lambda) \tag{9}$$

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[3]   Cochrane and Sbordone (1988, p. 282–283) point out that C_t and Y_t are cointegrated if w_t is a stationary ARMA process, stationary about a linear trend, or stationary about a geometric trend of order less than R^{-1}, so that the sum in (4) converges. They also show C_t and Y_t are cointegrated for general difference stationary wage processes.

in which the true MPC parameter varies in the frequency domain since it is determined by the filter $(1 - \gamma R^{-1})/(1 - \gamma R^{-1}L)$ in (7). The coherence between consumption and income at high frequencies falls, since the transfer function of the filter is given by

$$\left[\frac{1 - \gamma R^{-1}}{1 - \gamma R^{-1}e^{-i\lambda}} \cdot \frac{1 - \gamma R^{-1}}{1 - \gamma R^{-1}e^{i\lambda}}\right]^{1/2} = \left[\frac{(1 - \gamma R^{-1})^2}{1 + (\gamma R^{-1})^2 - 2\gamma R^{-1}\cos\lambda}\right]^{1/2} \tag{10}$$

which equals 1 evaluated at $\lambda = 0$ and is decreasing in λ over the interval $[0, \pi]$ provided $1 > \gamma > 0$.[4] This result may be considered a frequency domain version of Friedman's PIH.[5]

III   Econometric Methodology

The co-integrated system embodied in equations (5) and (6) is analogous to the general set-up in Phillips (1991, equations (1) and (2)). Phillips develops a spectral regression estimator for co-integrated systems based upon a triangular ECM representation of the co-integrated system. The estimator, appropriately modified, may also be applied to the levels of the co-integrating regression itself. We use the levels approach in this paper.

In order to use Phillips' estimator, we first transform (7) to frequency domain format by taking discrete Fourier transforms

$$\omega_c(\lambda) = \beta(\lambda)\omega_y(\lambda) + \omega_u(\lambda) \tag{11}$$

where

$$\beta(\lambda) = \beta , \qquad \omega_c(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} c_t e^{it\lambda}$$

$$\omega_y(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_t e^{it\lambda} , \qquad \omega_u(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} u_t e^{it\lambda}$$

for $\lambda \in [-\pi, \pi]$. Note that we have assumed $\beta(\lambda) = \beta$ for $\lambda \in [-\pi, \pi]$; this assumption is not necessary and is relaxed below. Phillips' estimator relies on an

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[4]   If $\gamma < 0$, then while the mpc is unity at zero frequency, the mpc is increasing over the interval $[0, \pi]$.

[5]   If we had assumed that disposable labor income was governed by the $AR(1)$ process $(1 - \gamma L)w_t = \varepsilon_t^w$, we would have retained the cointegration of C_t and Y_t as well as the analogue to (7):

$$C_t = \left(\frac{(1 - R^{-1})(1 - \gamma L)}{1 - \gamma R^{-1} - (\gamma + R^{-1} - 2\gamma R^{-1})L}\right)Y_t + \left(\frac{(1 - \gamma R^{-1})(1 - L)}{1 - \gamma R^{-1} - (\gamma + R^{-1} - 2\gamma R^{-1})L}\right)\varepsilon_t^c \tag{8}$$

The filter in (7) under the $AR(1)$ assumption also has the property that it equals 1 at zero frequency and is decreasing in λ over the interval $[0, \pi]$.

estimate of the spectrum of $v = [u, \Delta y]'$. We may use the residuals from an initial Hannan (1963) efficient single equation regression on (11) to estimate u. The spectrum of v may then be estimated by a variety of methods, e.g. by smoothing the periodogram, viz:

$$\hat{f}_{vv}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_v(\lambda_s)\omega_v(\lambda_s)^*$$

where the summation is over the frequency band B_j given by

$$\lambda_s \in B_j = \left(\theta_j - \frac{\pi}{2M} < \lambda \leq \theta_j + \frac{\pi}{2M} \right)$$

Minimization of the Hermitian form

$$\sum_B tr([\omega_v(\lambda_s)\omega_v(\lambda_s)^*]\Phi(\lambda_s))$$

with respect to β, where $\Phi(\lambda_s) = \hat{f}_{vv}(\omega_j)^{-1}\forall\lambda_s \in B_j$, leads to the following estimator:

$$\hat{\beta} = \left[\frac{1}{2M} \sum_{j=-M+1}^{M} e'\hat{f}_{vv}^{-1}(\theta_j)e\hat{f}_{yy}(\theta_j) \right]^{-1} \left[\frac{1}{2M} \sum_{j=-M+1}^{M} \hat{f}_{y*}(\theta_j)\hat{f}_{vv}^{-1}(\theta_j)e \right] \tag{12}$$

where

$$\hat{f}_{yy}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_y(\lambda_s)\omega_y(\lambda_s)^*$$

$$\hat{f}_{y*}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_y(\lambda_s)\omega_{[c, \Delta y]}(\lambda_s)^*$$

and $e' = (1, -\gamma)$. Note that if there is no coherence between u and Δy at frequency zero, the spectrum of v is diagonal for $\lambda \in [-\pi, \pi]$, and Phillips' estimator would be equivalent to the Hannan efficient single equation estimator if the true error spectrum were employed in (12). We also use estimates of the marginal propensity to consume which rely only on spectral estimates at the origin. This estimator is given by

$$\hat{\beta}(0) = \frac{\hat{f}_{yy}(0)^{-1}\hat{f}_{y*}(0)\hat{f}_{vv}^{-1}e}{e'\hat{f}_{vv}^{-1}(0)e} \tag{13}$$

Phillips (Theorem 3.1) proves that the limit distributions of $\hat{\beta}$ and $\hat{\beta}(0)$ are the same normal mixture distributions, and that the estimates are asymptotically median unbiased, and fully efficient under Gaussian errors. Furthermore, we can conduct tests of the hypothesis of constancy of the mpc across frequencies, or more general hypotheses about the co-integration space such as:

$$H_0: h(\beta) = 0 \ , \qquad H_1: h(\beta) \neq 0$$

where $h(\ )$ is a twice continuously differentiable function of restrictions on β. To test H_0 against H_1 we may employ the Wald statistic in its usual form. Thus, for the estimator $\hat{\beta}$ we would form

$$X_1 = h(\hat{\beta})'[\hat{H}V_T\hat{H}']^{-1}h(\hat{\beta})$$

where

$$\hat{H} = H(\hat{\beta}) \quad and \quad V_T = \frac{1}{T}\left[\frac{1}{2M}\sum_{j=-M+1}^{M} \gamma'\hat{f}_{vv}^{-1}(\theta_j)\gamma\hat{f}_{yy}(\theta_j)\right]^{-1}$$

Here V_T is the conventional estimate of the asymptotic variance matrix of $\hat{\beta}$ from spectral regression theory (see Hannan (1970), p. 442). Phillips proves that the test statistic X_1 converges asymptotically to the chi square distribution with degrees of freedom $= 1$.

When β changes with λ, an approximation to the frequency domain consumption function given in the time domain may be obtained as follows. By inversion of (9) we have:

$$C_t = \int_{-\pi}^{\pi} e^{it\lambda}\beta(\lambda)dZ_Y(\lambda) + u_t \ , \qquad where \quad u_t = \int_{-\pi}^{\pi} e^{it\lambda}dZ_u(\lambda) \tag{14}$$

Assuming that the frequency dependent mpc $\beta(\lambda)$ obeys the step function:

$$\beta(\lambda) = \begin{cases} \beta_1 \ , & \lambda \in B_1 = [-\pi/3, \pi/3] \\ \beta_2 \ , & \lambda \in B_2 = [-\pi, -\pi/3] \cup [\pi/3, \pi] \end{cases} \tag{15}$$

then (14) has the simple time domain form:

$$C_t = \beta_1 Y_t + u_t + \int_{-\pi}^{\pi} e^{it\lambda}(\beta(\lambda) - \beta_1)dZ_C(\lambda)$$

or

$$C_t = \beta_1 Y_t + \eta_t \ , \qquad where \quad \eta_t = u_t + \int_{B_2} e^{it\lambda}(\beta_2 - \beta_1)dZ_y(\lambda) \tag{16}$$

Equation (16) is a co-integrated system between C_t and Y_t since η_t is stationary. It follows that Phillips' spectral regression approach may still be used to estimate β_1. Moreover, restricting the regression to the lower frequencies will affect the asymptotic properties of the estimate of β_1 since η_t is stationary.

Estimation of the high frequency parameter, β_2, is more involved. One approach is to estimate β_2 by restricting the spectral regression to the high frequencies. However, this is unlikely to yield satisfactory estimates for β_2 since η_t and Y_t, though stationary over the high frequencies, will be coherent and this results in simultaneous equations bias. Since the limit variates for $\lambda \in B_2$ all have finite variance and $\omega_Y(\lambda)$ and $\omega_u(\lambda)$ are correlated in the limit, we need to instrument $\omega_Y(\lambda)$ to get consistent estimates of β_2. For example, if the generating mechanism for ΔY is $\omega_{\Delta Y}(\lambda) = \delta\omega_Z(\lambda) + \omega_\zeta(\lambda)$ and Z_t is independent of ζ_t and η_t, we can use $\omega_Z(\lambda)$ as an instrument in a spectral regression on (16). For the purpose of the simulations we shall use a frequency domain Generalized Instrumental Variable Estimator (GIVE). This estimator is based on the following formula, which may be interpreted as a spectral version of Sargan's (1989, p. 63) GIVE estimator:

β^{GIVE}

$$= \frac{[[\sum_{\theta_j \in B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)][\sum_{\theta_j \in B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)]^{-1}[\sum_{\theta_j \in B_2} f_{zc}(\theta_j)f_{uu}^{-1}(\theta_j)]]}{[[\sum_{\theta_j \in B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)][\sum_{\theta_j \in B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)]^{-1}[\sum_{\theta_j \in B_2} f_{zy}(\theta_j)f_{uu}^{-1}(\theta_j)]]}$$

(17)

with asymptotic variance matrix:

$V(\beta^{GIVE})$

$$= \frac{2M}{T}\left[\left[\sum_{\theta_j \in B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)\right]\left[\sum_{\theta_j \in B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)\right]^{-1}\left[\sum_{\theta_j \in B_2} f_{zy}(\theta_j)f_{uu}^{-1}(\theta_j)\right]\right]^{-1}$$

In both cases, feasible estimates are obtained by employing estimated spectra in these formulae. Note that (17) is based on frequencies in the band B_2 and is therefore a band spectral GIVE estimator for the parameter β_2 in (9) and (15).

IV Simulations

We now report the results of simulations designed to assess the merit of Phillips' systems procedure (henceforth denoted by SYS) relative to single equation band spectral regression (denoted by SNG). Our evaluation will be based on two criteria: (a) whether the estimator yields t-statistics with the correct size (given a nominal significance level); and (b) power.

Let $\omega_Y(\lambda) = (2\pi T)^{-1/2}\Sigma e^{it\lambda}Y_t$ denote the discrete Fourier transform of Y_t. The data generation process we employ may be represented as[6]:

$$\omega_C(\lambda) = \beta(\lambda)\omega_Y(\lambda) + \omega_{u1}(\lambda)$$

where

$$\omega_{\Delta Y}(\lambda) = \omega_{u2}(\lambda)$$

$$\beta(\lambda) = \begin{cases} 1.00 , & \lambda = [0, \pi/3] \\ 0.25 , & \lambda = (\pi/3, \pi] \end{cases}$$

and

$$Y_t = Y_{t-1} + u_{2t} , \qquad t = 1, 2, 3, \ldots, T$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = u_t = \xi_t + \psi\xi_{t-1} , \qquad \xi \sim N(0, \Sigma)$$

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[6]   We chose $\pi/3$, or six quarters, as the cutoff for the low and high bands in the empirical section. All the results are robust to alternative choices.

$$\psi = \begin{bmatrix} 0.3 & 0.4 \\ \psi_{21} & 0.6 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{21} & 1 \end{bmatrix}$$

Following Phillips and Loretan (1991), we consider values of $\{0.8, 0.4, 0.0, -0.8\}$ for ψ_{21} and $\{-0.85, -0.5, 0.5\}$ for σ_{21}. Note that the error term, u_1, is coherent with u_2 at frequency zero for every combination of σ_{21} and ψ_{21}. Thus the distribution of the SNG estimator should suffer from second-order bias (see Phillips (1991)).

Let $\hat{\beta}_i(0)$ (i = SYS, SNG) denote the parameter estimate for the true β over the low frequencies (i.e. $\lambda = 0$) and t ($\beta_i = \beta$) the t-statistic for the null hypothesis that $\beta_i(0) = \beta$. Column 2 of Table 1 (Part A) contains the variance ratio of SYS/SNG for $\beta(0)$. Column 3 reports the bias ratio of SYS to SNG. Column 4 and 5 report the mean bias in the two estimators. Size computations are reported in columns 6 and 7, and power computations are reported in columns 8 and 9. The size computations are based on the t-statistic for the null hypothesis that the low frequency $\beta = 1$ (using a 5 per cent level of significance). Power is computed by changing the true value to $\beta = 0.95$. All the simulations were conducted for $M = 2$, a sample size of 256, and 10,000 replications[7].

The results in Table 1 (Part A) suggest that the SYS estimator is superior to the SNG estimator since the SYS estimator consistently yields a smaller variance and bias than the SNG estimator. The simulations demonstrate that the variance ratio falls as the coherence between u_1 and u_2 increases. This is to be expected since the asymptotic variance of the SYS estimator depends on the conditional variance of u_1 given u_2. The efficiency gains can be large (see model C, F, and, in particular, J). The SYS estimator typically dominates the SNG estimator in terms of power across the 12 simulations. Interestingly, both estimators suffer from little size distortion. This suggests that the second-order bias effect due to the coherence of u_1 and u_2 does not have a great impact on the location of the finite sample distribution (at least for the parameter combinations chosen in these simulations).

Part B of Table 1 reports the results of applying the spectral regression estimator over the frequency band $[\pi/3, \pi]$. Column 2 contains the bias in the estimator, while column 3 presents empirical size. The results clearly demonstrate the simultaneity bias arising from the coherence of u_1 and u_2 over the high frequency. Except for model F, where the coherence over the high frequency is negligible, the level of bias is substantial. These simulations underscore the importance of using GIVE over the high frequencies.

The results of applying GIVE to the same model are presented in columns 4 and 5 of Table 1 (Part B), using an instrument which has zero coherence with u_1 (the true error term) for all frequencies. The instrument was generated by (recursively) accumulating the residuals from a regression of u_2 on u_1, which is

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Table 1. PART A: Monte Carlo results for model 1 (low frequency), $T = 256$ (10,000 replications)

Model	Var Ratio (SYS/SNG)	Bias Ratio (SYS/SNG)	Bias Level		Size		Power at $\beta = 0.95$	
			SYS	SNG	SYS	SNG	SYS	SNG
(A)	0.8745	0.1719	-0.3350E-04	-0.1948E-02	5.43	4.43	94.90	92.37
(B)	0.8951	-0.061	-0.7911E-06	0.1277E-02	5.12	4.69	97.97	98.15
(C)	0.2292	0.0044	0.1429E-06	0.3227E-02	5.23	1.57	100.00	99.91
(D)	0.6938	0.1209	-0.2939E-03	-0.2430E-02	4.26	2.92	99.10	97.01
(E)	0.9827	0.2385	0.7158E-06	0.3001E-03	5.81	5.47	97.47	97.51
(F)	0.3565	-0.022	-0.7381E-06	0.3325E-02	6.00	2.25	99.89	99.72
(G)	0.5735	0.1218	-0.2770E-03	-0.2773E-02	3.74	2.35	99.90	92.27
(H)	0.9472	0.1354	-0.9990E-06	-0.7377E-03	5.22	4.52	98.70	98.32
(I)	0.5361	0.0025	0.8250E-07	0.3323E-02	5.66	2.80	98.90	98.18
(J)	0.4839	0.1182	-0.2095E-03	-0.1771E-02	1.90	1.20	99.99	99.95
(K)	0.7699	0.1949	-0.3024E-03	-0.1551E-02	4.80	3.50	99.92	99.76
(L)	0.9331	0.0495	0.6894E-06	0.13901E-02	5.75	5.04	92.19	92.79

LEGEND

model code	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
σ_{21}	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50
ψ_{21}	0.80	0.80	0.80	0.40	0.40	0.40	0.00	0.00	0.00	-0.80	-0.80	-0.80
Coherence of u_1, u_2 at zero frequency	-0.36	0.30	0.88	-0.59	0.02	0.81	-0.72	-0.21	0.68	-0.83	-0.52	0.22

Table 1. PART B: Monte Carlo results for model 1 $(\pi, \pi/3)$, $T = 256$ (10,000 replications)

Model	SNG		SPECTRAL GIVE		
	Bias	Size	Bias	Size	Power
(A)	-0.2997	76.1	0.3654E-02	2.62	90.8
(B)	0.2197	66.9	0.5830E-02	2.50	95.0
(C)	0.6504	99.0	0.4392E-02	3.26	100.0
(D)	-0.4131	95.6	0.4419E-02	2.96	99.1
(E)	0.0219	5.30	0.9396E-02	2.78	94.6
(F)	0.6729	99.5	0.6641E-02	3.18	99.8
(G)	-0.4234	97.8	0.4526E-02	3.40	100.0
(H)	-0.1306	37.8	0.5291E-06	2.76	97.6
(I)	0.6102	98.5	0.4173E-02	3.06	97.4
(J)	-0.3689	99.3	0.3752E-02	3.72	100.0
(K)	-0.2497	90.0	0.5404E-02	3.06	99.9
(L)	0.2223	46.3	0.6887E-02	2.18	82.5

LEGEND

model code	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
σ_{21}	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50
ψ_{21}	0.80	0.80	0.80	0.40	0.40	0.40	0.00	0.00	0.00	-0.80	-0.80	-0.80
Coherence of u_1, u_2, at zero frequency	-0.36	0.30	0.88	-0.59	0.02	0.81	-0.72	-0.21	0.68	-0.83	-0.52	0.22

obviously orthogonal to u_1. The size of the correlation between u_2 and the instrument is inversely proportional to the covariance between u_2 and u_1. GIVE clearly overcomes the bias problem. However, the size of the t-statistic is consistently conservative; that is, less than the nominal setting of the test, which is 5% in this case. The power of the statistic is satisfactory.

V Empirical Results

This section provides a statistical analysis of U.S. aggregate consumption and income data. All data are seasonally adjusted quarterly series for the period 1948:1 to 1993:2, and are taken from the National Income and Product Accounts in Citibase. The series examined are per capita non-durable goods and services consumption and per capita disposable income expressed in 1987 dollars.

Table 2 provides tests of the unit root hypothesis for detrended consumption and disposable income using Phillips' (1987) Z_α and Z_t statistics and the Park and Choi (1988) $G(p, q)$ statistics. For the Z_α and Z_t statistics, the maintained hypothesis is a unit root. For the $G(p, q)$ statistic, the null hypothesis is stationarity around a p'th order time polynomial; the statistic possesses (asymptotically) a χ^2_{q-p} distribution. The statistical results imply that detrended con-

Table 2. Unit root tests for non-durable consumption and disposable income

	Levels			First Differences		
	Z_t	Z_α	$G(1,3)$	Z_t	Z_α	$G(0,3)$
C p-value Y p-value	-2.8635 -2.2667	-8.4363 -11.0128	31.5892 (0.00) 26.9988 (0.00)	-9.9577 -14.9804	-134.7706 -204.4946	7.7948 (0.05) 3.2796 (0.35)

Notes: The computed Z_t, Z_α, and $G(p, q)$ statistics are based on 5 lags. Levels statistics are based on detrended data with critical values (5%): $Z_t = -3.4229$; $Z_\alpha = -21.1196$. Differences are based on demeaned data with critical values (5%): $Z_t = -2.8759$; $Z_\alpha = -13.8070$

Table 3. Cointegrating regression: $C_t = \kappa + \gamma t + \beta Y_t$

κ	γ	β	Z_t	Z_α
834.452 (7.364)	3.8049 (3.293)	0.6862 (30.740)	-4.7667	-36.5187

Notes: The Z statistics are based on 5 lags. T-statistics are reported in the parentheses. Critical values (5%) for $Z_t = -3.9157$, $Z_\alpha = -25.5105$

sumption and income possess a unit root. First differences of the detrended data appear to be stationary (as required).

Table 3 presents OLS estimates of the co-integrating regression between consumption and income. This table also reports the results of applying the Z_α and Z_t statistic to the estimated residuals of the co-integrating regression. Both the Z_α and the Z_t statistics are significant at the 5% level of significance, thus providing support for the alternative hypothesis of co-integration.[8]

Table 4 presents band spectral estimates of the marginal propensity to consume using Phillips' systems procedure (SYS) and our generalized instrumental variable estimator (GIVE), comparing it to the single equation technique (SNG) when appropriate. The first panel provides estimates for the marginal propensity to consume using equation (13) $[\hat{\beta}(0)]$, with 46 periodogram ordinates to estimate the spectrum at frequency zero. The second panel provides estimates of the marginal propensity to consume over the restricted high frequency interval $[\pi/3, \pi]$, partitioning it into bands based on $M = 2$. These estimates are from the SNG and GIVE estimators, in the latter case using lagged real per capita disposable income, real per capita government expenditure, and the monetary base as instruments for real per capita disposable income.

Using the SYS estimator, the point estimate for the long-run mpc is approximately 0.7.[9] We can easily reject the null hypothesis that the long-run mpc is unity at the 5% level of significance. We obtain point estimates of 0.1645 and 0.1069 for the mpc over the high frequency band $([\pi/3, \pi])$, using SNG and GIVE estimators respectively. These estimates are significantly lower than the

Table 4. Spectral estimates of the marginal propensity to consume

Zero Frequency Estimator $\beta(0)$: SYS 0.7045 t-stat from $1 = -7.156$ SNG Frequency $(\pi/3, \pi)$ Estimator β: Levels 0.1645 t-stat from $0 = 1.961$	Zero Frequency Estimator $\beta(0)$: SNG 0.7039 t-stat from $1 = -7.0412$ GIVE Frequency $(\pi/3, \pi)$ Estimator β: Levels 0.1069 t-stat from $0 = 0.884$
$H_0: \beta(0) = \beta(\pi/3, \pi), H_1: \beta(0) \neq \beta(\pi/3, \pi)$ $\chi^2(1) = 5.7719$	

Notes: SYS denotes Phillips' (1991) systems spectral estimator while SNG denotes Hannan's (1963) single equation estimator as employed in Engle (1974). GIVE uses lagged real disposable income, real per-capita government expenditure, and the monetary base as instruments for real per-capita income. Critical value (5%) for the chi square test equals 3.84

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[8]   We note that the results do not depend on the choice of q. We were not able to reject the null hypothesis of cointegration for $q = 3, 4$, and 5 using a 5 percent level of significance.

[9]   Note that the SYS and SNG estimators produced very similar estimates for the long-run mpc. This is due to the fact that the residuals of the co-integrating regression and the first difference of disposable income have negligible coherence over $[-\pi, \pi]$. The efficiency gains in moving from SNG to SYS in this particular application are small.

zero frequency estimate, and in the GIVE case the estimate is insignificantly different from zero. The statistical results for the test of constancy of the MPC over the two frequency domain intervals are also provided in Table 4. We may perform a formal statistical test as outlined in Section III. The computed χ_1^2 statistic for the null hypothesis of parameter constancy is 5.7719, which exceeds the critical value of 3.84 at the 5% level. Thus we reject the null of constancy of the MPC across frequencies.

These results provide strong support for the declining mpc restriction of the PIH. It suggests that transitory disposable income innovations are saved (or dissaved), in contrast to many previous time domain tests (in particular, the results of Flavin (1981)). It also stands in contrast to the frequency domain test of Engle (1974). Besides the bias that may arise in the SNG estimator due to coherence between the error and the regressor suggested in our Monte Carlo study, perhaps the more important difference between our findings and that of Engle is the potential leakage problem associated with a deterministic trend.[10]

VI Conclusions

This paper has investigated the properties of the consumption function in the frequency domain. The PIH implies a system of two equations, one of which is a co-integrating regression between consumption and income and the other is an equation of motion for income. We examined this model in the frequency domain and its implications for the marginal propensity to consume at both low and high frequencies. The theoretical model implies that the MPC is unity at the zero frequency and that the MPC falls as we move to higher frequencies. New frequency domain estimates of the long run mpc which account for stochastic trends in aggregate consumption and disposable income data, as well as their joint dependence, are provided. The hypothesis that the long-run (or zero frequency) MPC is unity can be rejected in this dataset. However, the null hypothesis of constancy of the MPC across frequency bands may also be rejected. Thus, our empirical results provide some support for the PIH in the frequency domain.

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[10] That is, the high frequency mpc estimate may be distorted because of leakage in the periodogram from the low frequencies to the high frequencies. Given the unit-root behavior of the data, leakage from the low frequency ordinates to the high ordinates can be large, resulting in poor estimates of the high frequency mpc. One solution is to filter the data by taking first differences and re-estimate the model. Note that the spectral GIVE estimator is still consistent in this filtered model provided the instruments are valid. Another approach is to detrend the data. The leakage problem does not occur in our Monte Carlos since we generated the simulated data such that the actual frequencies are mapped one-for-one to the frequencies used by the discrete Fourier transform. The discretization of the actual (continuous) consumption data results in leakage from one frequency to another, and impairs the performance of the estimator, which is acute at the high frequencies where super consistency does not apply.

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