1

Rex Bergstrom's Career and Research

Peter C. B. Phillips

Rex Bergstrom* was trained as an economist in New Zealand, earning his B.Com and M.Com degrees in 1946 and 1948 at Canterbury University College, which at that time was a college of the University of New Zealand. He took up his first academic position as an Assistant Lecturer in Economics in 1948 at Massey College in Palmerston North. In 1950 he moved to Auckland University College where he worked until 1962, rising to the rank of Associate Professor of Economics. During the 1950s, Bergstrom became recognized as New Zealand's foremost econometrician, publishing his first applied econometrics article [6] in the *Economic Record* in 1951 and several other papers [5, 7, 8, 11, 12] on empirical aspects of the New Zealand economy over this period. The *Economic Record* paper developed and estimated an export supply function for the New Zealand economy and was the precursor to his famous model of supply and demand for New Zealand exports that was published by *Econometrica* in 1955. The *Econometrica* article [8] was based on Bergstrom's doctoral dissertation at Cambridge, which he had completed while on leave from Auckland during 1952–4, supported by a Travelling Scholarship in Commerce. The article quickly attracted international attention. It reported the largest system of simultaneous equations ever formulated for empirical work up to that time; it was estimated by the new limited-information maximum likelihood (LIML) methodology devised by the Cowles Commission; and its empirical results gave strong support for the new methodology. The skilful mix of economic theory, econometric methodology, and painstaking empirical implementation that was demonstrated in this article became the hallmark of Bergstrom scholarship.

By the end of the 1950s Bergstrom's interests in econometrics had widened considerably beyond empirical research. In search of a more complete understanding of the properties of the LIML procedure that he had used so successfully in his empirical model of New Zealand exports, Bergstrom set about finding the exact finite-sample distributions of the maximum likelihood estimator (MLE) and ordinary least squares (OLS) estimator of the propensity to consume in a simple stochastic income determination model. The Indian econometrician Nagar (1959) had earlier

*References to Bergstrom's articles are in square brackets [ ] and refer to the bibliography of his work in chapter 4.

developed moment approximations for simultaneous equations estimators and his study provided the immediate catalyst for Bergstrom's research. Bergstrom's work turned out to be very different from Nagar's. He derived exact mathematical forms for the density functions of the MLE and OLS estimators, graphed the densities and computed probabilities of concentration about the true value of the parameter for the two estimators. The results were conclusive and provided clear support for the use of simultaneous equations estimators like LIML. Bergstrom's results were published by *Econometrica* in 1962. Remarkable as a first piece of technical research, Bergstrom's paper [13] ushered in a new era of mathematical sophistication in econometrics. Independently, Basmann (1961) had published some related work on the exact density of the two-stage least squares estimator and together their articles are recognized as having opened up a new research field.

In 1962 Bergstrom left Auckland for the London School of Economics (LSE) where he became a Reader in Economics. His arrival coincided with the time when the LSE was rising to prominence in the field of econometrics and overtaking Cambridge as the leading center of econometric research in the UK. Bergstrom joined a burgeoning group of econometricians that included another New Zealand born economist, Bill Phillips, the statistician Jim Durbin and, somewhat later, Denis Sargan, as well as a larger team of economists such as Dick Lipsey, Chris Archibald and Morris Peston all of whom were working vigorously in applied econometrics. The initiatives established during this period at the LSE were to put the School at the forefront of econometric research in the UK for the next two decades.

During his period at the LSE from 1962–4, Bergstrom's own research on econometric modeling moved in decisive new directions. Influenced by the earlier work of Bill Phillips, he embarked on a research agenda concerned with the development of economic models of cyclical growth that synthesized real and monetary phenomena in a growing economy. These models, which were published in *Economica* in 1962 [14] and *Economic Studies Quarterly* in 1966 [17], incorporated a neoclassical production technology, Keynesian-type feedbacks from prices and interest rates to the real sector and adjustment mechanisms to accommodate market disequilibrium. As economic models, they represented an important advance over earlier trade cycle models which set out to explain cyclical behavior in real variables through the multiplier–accelerator mechanism alone. In addition, the new models had particular solutions that corresponded to the steady state paths of the earlier neoclassical growth models of Solow (1956) and Swan (1956). The Bergstrom cyclical growth models therefore provided a synthesis of two earlier strands of research and were the first truly disequilibrium models of neoclassical growth.

The formulation of cyclical growth models as ordinary differential equations with a continuous time parameter leads to econometric specifications as systems of stochastic differential equations. Apart from some isolated early research by Bartlett (1946), Quenouille (1957), Phillips (1959), and Durbin (1961), econometric methodology for estimating systems of stochastic differential equations was completely undeveloped at this time. During his period at the LSE, Bergstrom commenced an ambitious research program that was complementary to his work on economic models of cyclical growth. This research program explicitly addressed the estimation of econometric models in continuous time with discrete data. Bergstrom's research on this topic was motivated by concerns of practical implementation and led to his proposal of nonrecursive discrete approximations to continuous-time systems. The approximations were to be constructed in the form of simultaneous equations models and the proposal had the natural advantage of making available the by-then well-established econometric methodology of simultaneous equations for estimation and inference in continuous-time models formulated as systems of differential equations. The discrete approximation is especially interesting because it shows the precise sense in which a system of simultaneous interdependent equations may be regarded as an approximation to a recursive differential equation system. The approach was systematically explored in an article [16] published in *Econometrica* in 1966 and it laid the foundation for much later empirical econometric work with continuous-time models.

The twin themes of economic models of cyclical growth and econometric estimation in continuous time have remained a major force in Bergstrom's thinking since the 1960s. In 1964 Bergstrom returned to Auckland University as Professor of Econometrics, where he completed his monograph [1] on *The Construction and Use of Economic Models*, published by the English University Press in 1967. This monograph is a masterful synthesis of earlier developments in trade cycle and cyclical growth models and it gives full expression to the disequilibrium modeling methodology that forms the basis of Bergstrom's later research. Indeed, the monograph develops a single-sector disequilibrium growth model that is the prototype for his subsequent empirical work on modeling the UK economy. In its final chapter, this monograph provides a beautifully succinct description of simultaneous equations econometric methodology and its application to the estimation of discrete approximations to stochastic differential equation systems.

In 1970 Bergstrom went back to England as Keynes Visiting Professor at the University of Essex, taking up a permanent Professorship there the following year. Administrative appointments at the senior University level, first as Dean and later as Pro-Vice-Chancellor, as well as a term as Departmental Chairman all reduced Bergstrom's time for research during the ensuing decade. Nevertheless, it was over this period that two of his most significant research accomplishments were completed.

The first of these is a major empirical study of the UK economy based on a continuous time econometric model of disequilibrium growth. This study [23], which was co-authored with Clifford Wymer (a former student from Auckland University who was at that time on the faculty of the LSE), was completed in 1974 and constitutes one of the most innovative empirical econometric exercises ever undertaken up to that time. The Bergstrom–Wymer model (as it soon became known), is a 13-equation continuous-time stochastic system that is based on a formal one-sector model of a macroeconomy. It is heavily constrained by cross-equation restrictions delivered from an underlying neoclassical growth theory. The model is estimated with postwar quarterly data by Gaussian techniques, its long-run properties are examined analytically, a stability analysis is conducted, and impressive out-of-sample forecasting performance is demonstrated. This model is distinguished from other empirical macromodels in many ways but most notably because it has only one exogenous variable, a simple time trend, and performs so well in empirical exercises in spite of this major simplification. The Bergstrom–Wymer model is now a classic study in empirical econometric research. It is a showpiece of careful applied econometrics and is the prototype for many similar models that have since been developed for other countries.

Bergstrom's second major line of research at Essex began in the early 1980s as a theoretical study of Gaussian estimation in continuous-time systems. By the end of the decade it had matured, like his earlier research on discrete approximations, into a complete econometric methodology that involved a theoretical development of inference, computational algorithms, computer programs, and empirical applications. Several doctoral students who worked under Bergstrom's supervision at Essex during the 1980s have helped in the completion of this major research program.

The first step in the new program was the econometric theory. Some earlier work by the author (1972) that was done under Bergstrom's supervision at Auckland University in 1969–70 had indicated that there were definite advantages in terms of bias reduction and efficiency gains from utilizing the exact discrete-time model of a continuous-time system for econometric estimation instead of the discrete approximation. In 1983, Bergstrom published in *Econometrica* an extensive study [25] of exact discrete-time models corresponding to stochastic differential equation systems. This paper broke new ground by allowing for higher order systems of equations and the presence of both stock and flow variables. The form of the exact discrete-time model that accommodated these complexities was found to be a vector autoregressive moving average model (in fact, VARMA(2,2) for a second-order stochastic differential equation system) whose coefficient matrices were highly nonlinear functions of the original system's parameters. Gaussian estimation involved the construction and optimization of the Gaussian likelihood for this discrete-time model. The statistical properties of the resulting estimates then followed from existing asymptotic theory for the estimation of stationary vector time-series models in discrete time, such as that developed by Dunsmuir (1979).

The 1983 paper set the stage for much subsequent research. The next step in Bergstrom's research addressed computational issues. In practical work, the principal difficulty in the implementation of Gaussian maximum likelihood is the construction of the likelihood. The problem is acute when there are data irregularities such as mixed stock and flow variables, missing observations, or initialization on a state vector with unknown elements. Unfortunately, these problems arise regularly in continuous-system estimation. One general approach to the construction of the likelihood is the Kalman filter. The use of this method in continuous-system estimation has been studied recently in a series of articles by Harvey and Stock (1985, 1988a, 1988b). In two papers [29, 30] published in 1985 and 1986, Bergstrom explored another approach that does not rely on the Kalman filter but makes explicit use of the exact discrete model for the observable variables, including the functional dependence of the error covariance matrix (Ω) on the system parameters. The likelihood can then be constructed readily from the conventional Gaussian form, given an initialization of the state vector, by using the Choleski factorization of Ω. The resulting algorithm involves computations that are on the order of Tn^3, where n is the dimension of the system and T is the sample size. Computations of this order within a general algorithm to maximize a likelihood function are well within the capability of modern computing equipment, at least for moderate values of n and T.

With the development of a complete theory of Gaussian estimation of continuous systems and a computational algorithm to implement it, Bergstrom turned to address issues of forecasting, control, and statistical testing. In an article [33] published in *Computers, and Mathematics with Applications* in 1989, Bergstrom showed how to generate optimal forecasts for a continuous system in a simple way from the discrete VARMA model using the Gaussian estimates of the matrix Choleski factorization of the full error covariance matrix. Recursive formulas were provided for the generation of k-period ahead forecasts. A related article [3] dealt with optimal control and provided the solution of a quadratic cost functional optimization problem given stochastic differential equations of motion for the system. This work extended earlier certainty equivalence theory and allowed for weaker conditions on the innovation processes by relaxing the requirement that they be increments of Brownian motion.

Statistical testing and model evaluation of continuous systems form the subject of a very recent article [34] published by Bergstrom (1990) in a volume that brings together his contributions to continuous-time econometric modeling. The subject of [34] is the development of practical statistical tests of the models and explicit parameterizations that arise in discrete versions of continuous systems. Discrete data generated from a continuous system with mixed stock and flow variables satisfy a VARMA model with heavily restricted coefficient matrices. For a second-order system of stochastic differential equations with mixed stock and flow data, the VARMA model is of order (2,2). A further complication is that the MA coefficient matrices are temporarily dependent. [34] gives recursive formulas for the computation of these matrices and shows that the recursion is stable in the sense that the coefficients converge (as $t \to \infty$) to fixed limit matrices. Several statistical tests are suggested to evaluate the model. These involve three steps:

1. test the hypothesis that a VARMA(2,2) is the data generating mechanism by time-series model selection procedures;
2. test the form of the coefficient matrices in the VARMA(2,2) that are implied by an underlying continuous system using a likelihood ratio test;
3. test the explicit parameterization of the coefficient matrices that are implied by economic theory using a Wald test.

These tests permit an investigator to assess the adequacy of the maintained hypothesis (i.e. a VARMA(2,2) model) within which the continuous system's discrete model is embedded, to test the validity of the restrictions implied by the presence of an underlying continuous time system, and, finally, to test any restrictions delivered by economic theory on the coefficients in the continuous systems.

The article on statistical testing completed Bergstrom's decade-long study of the econometric methodology of higher order continuous-time systems. The empirical implementation of this new methodology is clearly the next phase of research and will carry Bergstrom's commitment to this line of research to its logical conclusion and into the 1990s. The empirical research program has now begun with a joint article [35] with M. J. Chambers (one of Bergstrom's former Ph.D students at Essex) on the demand for durable goods in the UK over the period 1973–84. This article studies dynamic responses of consumer purchases to changes in disposable income and post-sample predictive performance of the model is tested against simpler models. The results are deemed to be a successful empirical implementation of the continuous time econometric methodology.

The most ambitious empirical application of the Bergstrom methodology of Gaussian estimation of higher order continuous systems is to a new continuous-time macroeconometric model of the UK. At the time of writing, this application is nearing completion and is a joint enterprise with Ben Nowman and Clifford Wymer, two of Bergstrom's former students. It promises a major implementation of the latest econometric methodology for continuous-time systems and an important new extension of the earlier Bergstrom–Wymer empirical work on modeling the UK economy. In a very real sense this new empirical application will represent the culmination of a research program that has been ongoing for the last 30 years.

Since the early 1960s Bergstrom has been the world's leading proponent of continuous-time econometric modeling. Few econometricians have ever shown such tremendous dedication to a field of research. These days such dedication is rare even on much shorter time horizons, particularly in North America where funding agencies play a large role in influencing research directions and fads and fashion all too often seem to take the place of serious long-term research agendas. Bergstrom has established a tradition of research wherein a class of exciting economic theory models are formulated and analyzed, econometric methodology is developed, computational algorithms and computer programs are written to permit practical implementation of the methods, and empirical applications are conducted that explore the practical consequences and worldly implications of the new methodology. This must surely be econometrics at its most fully developed. As a paradigm for students and young researchers it is certainly the finest example. As a lifetime achievement in research it is remarkable.

References

Bartlett, M. S. (1946). "On the theoretical specification and sampling properties of autocorrelated time-series," *Journal of the Royal Statistical Society Supplement*, 8, 27–41.

Basmann, R. L. (1961). "A note on the exact finite sample frequency functions of generalized classical linear estimators in two leading overidentified cases," *Journal of the American Statistical Association*, 56, 619–36.

Bergstrom, A. R. (1990). *Continuous Time Econometric Modelling*. Oxford: Oxford University Press.

Dunsmuir, W. (1979). "A central limit theorem for parameter estimation in stationary vector time series and its application to models for a signal observed with noise," *Annals of Statistics*, 7, 490–506.

Durbin, J. (1961). "Efficient fitting of linear models for continuous stationary time series from discrete data," *Bulletin of the International Statistical Institute*, 38, 273–83.

Harvey, A. C., and J. H. Stock (1985). "The estimation of higher-order continuous time autoregressive models," *Econometric Theory*, 1, 97–117.

Harvey, A. C., and J. H. Stock (1988a). "Continuous time autoregressive models with common stochastic trends," *Journal of Economic Dynamics and Control*, 12, 365–84.

Harvey, A. C., and J. H. Stock (1988b). "Estimating integrated higher-order continuous time autoregressions with an application to money-income causality," *Journal of Econometrics*, 42, 319–36.

Nagar, A. L. (1959). "The bias and moment matrix of the general k-class estimators of the parameters in structural equations," *Econometrica*, 27, 575–95.

Phillips, A. W. (1959). "The estimation of parameters in systems of stochastic differential equations," *Biometrika*, 46, 67–76.

Phillips, P. C. B. (1972). "The structural estimation of a stochastic differential equation system," *Econometrica*, 40, 1021–41.

Quenouille, M. H. (1957). *The Analysis of Multiple Time Series*. London: Griffin.

Solow, R. M. (1956). "A contribution to the theory of economic growth," *Quarterly Journal of Economics*, 70, 65–94.

Swan, T. (1956). "Economic growth and capital accumulation," *Economic Record*, 32, 334–61.