TESTING FOR COINTEGRATION USING PRINCIPAL COMPONENTS METHODS*

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This paper studies cointegrated systems of multiple time series which are individually well described as integrated processes (with or without a drift). Necessary and sufficient conditions for cointegration are given. These conditions form the basis for a class of diagnostic statistical procedures designed to test for cointegration. The procedures rely on principal components methods. They are simple to employ and they involve only the standard normal distribution Monte Carlo simulations reported in the paper indicate that the new procedures provide simple and useful diagnostics for the detection of cointegration. Some empirical applications to macro-economic data are conducted and discussed.

1. Introduction

A recent development that seems likely to be of lasting importance to the statistical analysis of economic time series is the theory of cointegration. The idea of cointegrated variables was introduced by Granger (1981, 1983) and Granger and Weiss (1983) and has been more systematically studied in the recent paper by Engle and Granger (1987). Cointegrated systems allow individual time series to be integrated of order one, I(1), but require certain linear combinations of the series to be stationary or I(0). This framework accommodates rather well the empirical observation that individual economic time series often exhibit nonstationary characteristics but that certain combinations of the series tend to move together over time. The notion may also be regarded as a statistical embodiment of ideas from economic theory concerning long-run regularities or steady state behavior among economic variables. Examples now include modern theories of asset prices, purchasing power parity and the term structure of interest rates [see Campbell (1986), Corbae and Ouliaris (1987),

and Campbell and Shiller (1987)] as well as steady state theories of aggregate variables. The hypothesis of cointegration is therefore important in terms of its underlying economic ideas of long-run equilibrium and in terms of its statistical implications for applied research. Useful overviews of the subject have recently been written by Granger (1986) and by Hendry (1986).

Evidence that a cointegrating vector exists provides strong support for a long-run relationship among a group of variables whose short-run behavior may be very much more complex. However, detecting the existence of cointegration in a multiple time series seems to give rise to nonstandard testing procedures. This is because the asymptotic theory of regression in cointegrated systems is very different from conventional theory for stationary time series, as is clear from earlier work by Phillips (1986a) and Phillips and Durlauf (1986). These authors provide a detailed study of regression theory in the presence and absence of cointegration. In both cases the limiting distribution theory is nonstandard. Conventional significance tests and regression diagnostics have nuisance parameter dependencies even asymptotically and this complicates the use of the asymptotic theory for inference. Moreover, in the absence of cointegration the parameters of the system are unidentified [see Phillips (1987b)] and the estimated regression coefficients have nondegenerate limiting distributions. This further complicates the asymptotic theory for residual based diagnostic tests.

All of these complications present obstacles to the development of statistical tests of cointegration. Engle and Granger (1987) recommend the use of tests based on the residuals of cointegrating regressions. Under the null hypothesis of no cointegration all linear combinations of the variables are nonstationary. Under the alternative of cointegration, at least one linear combination (which may be consistently estimated by the cointegrating regression) is stationary. Test statistics for nonstationarity (or the presence of a unit root) in the residuals of the cointegrating regression might therefore be expected to provide discriminatory power against the (alternative) hypothesis of cointegration. Because of the complications discussed in the last paragraph, all of the statistics considered by Engle and Granger have nonstandard limiting distributions and these distributions are different from the usual limit distributions of simple tests for unit roots. The asymptotic properties of these residual based tests for cointegration have recently been studied in another paper by the authors [Phillips and Ouliaris (1987)].

It is possible to test for cointegration without using the residuals of a cointegrating regression. One alternative arises from the work of Phillips and Durlauf (1986). These authors explored multivariate tests for the presence of unit roots in multiple time series and gave a limiting distribution theory for Wald and modified Wald statistics under the null of no cointegration. They also constructed some general specification tests whose asymptotic distributions are χ^2, again under the null of no cointegration. Both procedures may be

used to test against the alternative of a cointegrated system and both yield consistent tests. The Phillips–Durlauf tests rely in a simple way on the estimated coefficient matrix in a first-order vector autoregression (VAR). Under the null of no cointegration this estimator is $O(T)$-consistent for the unit matrix. But, in a cointegrated system its probability limit is no longer the unit matrix. Hence, multivariate unit root tests may be expected to provide discriminatory power in the presence of cointegration. Stock and Watson (1986) have subsequently pursued this approach to the subject. Their paper explains the prefiltering of the data and the serial correlation corrections that are needed to remove parameter dependencies; and they recommend that attention be focused on the modulus of the smallest latent root of the regression coefficient matrix (of the VAR) in mounting a test of cointegration. In the scalar case their procedure reduces to the unit root test introduced in Phillips (1987a).

The main purpose of the present paper is to suggest a rather different approach to testing for cointegration. The intuition behind the procedures we develop is quite simple. In effect, our approach is to perform a form of principal components analysis for time series. When a multiple time series is cointegrated, the cointegrating vector effectively reduces the variability in the original series [which are taken to be I(1) processes] by an order of magnitude [from I(1) to I(0)]. This reduction in variance should be detectable by principal components methods.

More specifically, if multiple time series which are individually I(1) move together over time so that some combination of the series is I(0), then this implies restrictions on the innovations that drive the full system. Phillips (1986a) showed that a necessary condition for cointegration is that the spectral density matrix of the innovation sequence (which we take to be weakly stationary) has deficient rank at the origin. We call this matrix Σ. Moreover, the number of zero latent roots of Σ is the number of cointegrating vectors and the associated latent vectors of Σ are the cointegrating vectors themselves. This result suggests that we can test for cointegration by assessing whether or not Σ has a negligible latent root. Moreover, since nonparametric consistent estimates of Σ are easily obtained it is possible to develop simple asymptotic tests of the cointegration hypothesis which apply for a wide class of underlying innovations. To make matters even simpler, under general conditions consistent estimates of Σ and hence its latent roots are asymptotically normal. This indicates that diagnostic procedures can be developed which involve only the standard normal distribution.

This paper suggests two new procedures for detecting the presence of cointegration. The first test involves computing one-sided confidence intervals for the smallest latent root of Σ (or an associated correlation matrix P). The second test is similar but relies on the ratio of the smallest latent root to the sum of the latent roots of Σ (or P). We conduct a Monte Carlo study to assess

the properties of the new procedures. On the basis of the first stage of the Monte Carlo simulations, which compare a cointegrated system of integrated variables with a noncointegrated (or spurious) system of integrated variables, we recommend a simple rule for detecting cointegration amongst a group of integrated time series. This rule works well in our experiments for models of different dimensions and for a variety of data generating mechanisms. Also, in order to evaluate the ability of the rule to discriminate between structures which are cointegrated and those which are nearly cointegrated, we develop a model which allows one to systematically control the degree of cointegration. The simulation results indicate that the bounds procedure provides a useful diagnostic for detecting the absence of cointegration.

In the scalar case our procedures may be used as (autoregressive) unit root tests. But they are more directly interpretable as tests for the presence of a unit root in the moving average (MA) representation of a stationary time series. In effect, our tests may be regarded as tests for the invertibility of an MA representation. This hypothesis is itself of independent interest. We therefore hope that our procedures will have some applications in this context as well as that of cointegrated systems.

Our organization of the paper is as follows. Necessary and sufficient conditions for cointegration are given in section 2. Our bounds tests for cointegration are developed in section 3. We present several possible procedures, all centered on the same basic idea; and in section 4 we show how these results may be interpreted as tests of invertibility. The procedures are examined and compared in simulations that we report in section 5. Size and power comparisons are given for models of different dimensions and various plausible data generating mechanisms. Section 6 reports some empirical applications of our methods to macroeconomic data. Concluding remarks are made in section 7.

2. Conditions for cointegration

Let $\{y_t\}_0^\infty$ be a multiple $(n \times 1)$ time series that is generated in discrete time according to

$$y_t = Ay_{t-1} + u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

with

$$A = I_n,$$

and where y_0 may be any random vector, including a constant. In (1) $\{u_t\}$ is a zero mean, weakly stationary innovation sequence with spectral density matrix $f_{uu}(\lambda)$. Throughout the paper we shall require that

$$E|u_{i0}|^\beta < \infty \quad (\text{for} \quad i = 1, \ldots, n) \quad \text{and some} \quad \beta > 2, \tag{2}$$

and

$\{u_t\}_0^\infty$ is strong mixing with mixing numbers α_m

that satisfy $\sum\limits_{m=1}^{\infty} \alpha_m^{1-2/\beta} < \infty$. (3)

Under these conditions,

$$\Sigma = 2\pi f_{uu}(0) = \mathrm{E}(u_0 u_0') + \sum_{k=1}^{\infty} \mathrm{E}(u_0 u_k' + u_k u_0') \tag{4}$$

[Phillips and Durlauf (1986, corol. 2.2)]. In fact, the series defining Σ is absolutely summable in view of (2) and (3). Under these conditions, therefore, $f_{uu}(\lambda)$ is bounded and (uniformly) continuous on $[-\pi, \pi]$.

We now make explicit the hypothesis of cointegration. The variables of y_t are said to be cointegrated if there exists an n-vector $\gamma \neq 0$ for which $\gamma' y_t$ is stationary. More specifically, we shall define y_t to be cointegrated if there exists a vector $\gamma \neq 0$ for which $v_t = \gamma' y_t$ is weakly stationary with continuous spectral density. This ensures that the action of the cointegrating vector reduces the integrated process y_t to a stationary time series with properties analogous to those of the innovations driving the mechanism (1). It follows directly from (1) that

$$\gamma' u_t = v_t - v_{t-1}. \tag{5}$$

Thus, some combination of the innovations in (1) has an MA representation with a unit root. (Note that we are not asserting that v_t is white noise.) We deduce:

Theorem 1. The system (1) is cointegrated with cointegrating vector $\gamma \neq 0$ iff

$$\gamma' f_{uu}(\lambda)\gamma = c\lambda^2 + \mathrm{o}(\lambda^2) \quad as \quad \lambda \to 0 \tag{6}$$

for some constant c (possibly zero).

Proof. In view of (5) the spectrum of $\gamma' u_t$ is zero at the origin. To prove the necessity of (6) we observe that

$$\gamma' f_{uu}(\lambda)\gamma = |1 - e^{i\lambda}|^2 f_v(\lambda),$$

where $f_v(\lambda)$ is the spectrum of $v_t = \gamma' y_t$. Since $f_v(\lambda)$ is continuous under the

hypothesis of cointegration and since $|1 - e^{i\lambda}|^2 = \lambda^2 + O(\lambda^4)$ as $\lambda \to 0$, we deduce that for some constant c

$$\gamma' f_{uu}(\lambda)\gamma = c\lambda^2 + o(\lambda^2),$$

proving the necessity of (6). This condition is also sufficient because $f_v(\lambda)$ is continuous and bounded on every interval $\varepsilon \leq \lambda \leq 2\pi$, $\varepsilon > 0$ [since $f_{uu}(\lambda)$ is continuous and bounded]; and, in view of (6), $f_v(\lambda) \to c$ as $\lambda \to 0$. Hence, v_t has continuous and bounded spectrum and is weakly stationary. ∎

Note that (6) implies the necessary condition

$$\gamma' \Sigma \gamma = 0 \tag{7}$$

(that is, Σ is singular and γ lies in its null space). This necessary condition was given and discussed earlier in Phillips (1986a). When there are several distinct cointegrating vectors γ_i ($i = 1, \ldots, k < n$) we have $\Sigma \gamma_i = 0$ and Σ has k zero latent roots.

Condition (6) is necessary and sufficient. It is important in what follows because it more completely characterizes the properties of the spectrum $f_{uu}(\lambda)$ under the hypothesis of cointegration. In particular, it tells us that $\gamma' f_{uu}(\lambda)\gamma$ is not only zero at $\lambda = 0$ but flat at the origin as well. This means that for cointegrated systems such components of the spectrum should be well estimated by an average of the periodogram ordinates in a band centered on the origin.

3. Bounds tests of cointegration

We shall develop tests based on the latent roots of a consistent estimate of the covariance matrix Σ given in (4). These tests may be regarded as performing a form of principal component analysis in the frequency domain, a subject on which there is a large literature [see, for example, Brillinger (1981, ch. 9) and the references therein]. The latter methods are concerned with approximating a given multiple time series by another that is of lower dimension and yet contains much of the information of the original series. When a multiple time series is cointegrated there clearly exists a linear filter of the series which retains much of the variability of the original series but which is of lower dimension. The issue of practical importance is whether the variability that is lost by this reduction is small enough to be negligible. In a cointegrated system the lost variability is smaller by an order of magnitude (of integration) so it should be possible to make an empirical assessment of the existence of cointegration by principal component methods. The procedures we now develop are inspired by this line of reasoning.

We first consider the following estimator of Σ:

$$S_{Tk} = 2\pi \hat{f}_{uu}(0)$$

$$= 2\pi \left[\frac{1}{2k+1} \sum_{s=-k}^{k} I_{uu}\left(\frac{2\pi s}{T}\right) \right]$$

$$= \frac{2\pi}{2k+1} \left[I_{uu}(0) + \sum_{s=1}^{k} \text{Re}\left(I_{uu}\left(\frac{2\pi s}{T}\right) \right) \right], \tag{8}$$

where

$$I_{uu}(\lambda) = w_u(\lambda) w_u(\lambda)^*$$

is the periodogram, and

$$w_u(\lambda) = (2\pi T)^{-1/2} \sum_{1}^{T} u_t e^{i\lambda t}$$

is the finite Fourier transform. In practical work $I_{uu}(\lambda)$ can be computed using the fast Fourier transform for highly composite T, although the computation of (8) is in no way burdensome for typical sample sizes in economics.

S_{Tk} is a smoothed periodogram estimate of $\Sigma = 2\pi f_{uu}(0)$. It is consistent as $T \uparrow \infty$ provided $k \uparrow \infty$ in such a manner that $k/T \downarrow 0$. The associated matrix $\hat{f}_{uu}(0)$ in (8) is the Daniell estimate of the spectral density matrix at the origin [see, for example, Priestley (1981, pp. 440–441)] and this involves a rectangular spectral window. Of course, other choices of spectral window may be used, leading to alternative estimators of Σ. However, the flat behavior of $\gamma' f_{uu}(\lambda)\gamma$ in the vicinity of the origin in the presence of cointegration [see (6) above] indicates that the choice of a rectangular spectral window may be rather appropriate for the purpose we intend.

To make our approach as general as possible we shall often wish to allow for a drift in the generating mechanism (1). In this case (1) is replaced by

$$y_t = \mu + y_{t-1} + u_t, \qquad t = 1, 2, \ldots, \tag{1'}$$

with some constant n-vector μ. When there is cointegration in the system we have $\gamma' \mu = 0$, so that the cointegrating vector now annihilates the drift as well as the spectrum of u_t at the origin.

It is easy to accommodate (1') in our approach. We simply remove the zero frequency periodogram ordinate from (8) and continue to compute the finite Fourier transforms using first differences Δy_t. This is equivalent to computing

(8) using first differences about their fitted mean (i.e., $\Delta y_t - \overline{\Delta y}$). Adjusting for degrees of freedom in (8) we therefore recommend the use of the following estimate of Σ:

$$S_{Tk} = \frac{\pi}{k} \sum_{s=1}^{k} \mathrm{Re}\left(I_{uu}\left(\frac{2\pi s}{T} \right) \right), \qquad (9)$$

which we compute using measured first differences $u_t = \Delta y_t$.

Note that we do not suggest the use of regression residuals such as $\hat{u}_t = y_t - \hat{A} y_{t-1}$ in the estimation of Σ. The reason is that under the alternative hypothesis of cointegration the least squares coefficient matrix $\hat{A}$ does not converge in probability to the identity matrix [see Park and Phillips (1987)] and estimates of Σ that are based on $\hat{u}_t$ converge in general to a nonsingular matrix and are thereby inconsistent. The bounds procedures given below rely on consistent estimation of Σ (and hence its smallest latent roots) under the alternative of cointegration in order to obtain discriminating power between cointegrated and noncointegrated systems. We remark that the situation is different with residual based tests for cointegration for which consistent tests do rely on the use of regression residuals [see Phillips and Ouliaris (1987) for a discussion of this point].

As $T \uparrow \infty$ with k fixed we know from standard spectral theory [for example, Brillinger (1981, theorem 7.3.3)] that

$$S_{Tk} = \frac{\pi}{k} W_n(2k, f_{uu}(0)) \equiv \frac{1}{2k} W_n(2k, \Sigma), \qquad (10)$$

where $W_n(.\,,.)$ signifies a Wishart matrix of dimension $n \times n$ with degrees of freedom and covariance matrix given by the first and second arguments of W_n, respectively. The latent roots of S_{Tk} are correspondingly distributed as the latent roots of the scaled Wishart matrix in (10) when $T \uparrow \infty$ for k fixed [Brillinger (1981, theorem 9.4.4)]. Note that we use the symbol $\equiv$ in (10) to signify equality in distribution.

In view of Theorem 1 our main concern in testing for cointegration is naturally with the smallest latent roots of S_{Tk}. In particular, we need to assess whether these roots are negligible or statistically insignificant. This is, of course, a central element in principal components theory in multivariate analysis. Here the relevant distribution theory for the latent roots and extreme latent roots of a Wishart matrix has been fully developed in recent years. Muirhead (1982, ch. 9) provides an extensive review of these developments. Unfortunately, the distributions of the extreme roots depend on the full eigenstructure of the covariance matrix Σ and involve zonal polynomial representations which make computational work difficult. The conventional approach in principal components theory has therefore been to work with

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large sample approximations to the Wishart distribution $W_n(2k, \Sigma)$ for large k. Much of this theory was originally developed by Anderson (1963). When the latent roots of Σ are distinct the results are particularly simple. If l_i and λ_i $(i = 1, \ldots, n)$ are the latent roots of S_{Tk} and Σ, then l_i is asymptotically independent of l_j $(i \neq j)$ and the standardized variates,

$$k^{1/2}(l_i - \lambda_i)/\lambda_i, \qquad i = 1, \ldots, n,$$

are asymptotically $(k \uparrow \infty)$ $N(0,1)$ [see, for example, Muirhead (1982, p. 403)]. Now order the roots as $\lambda_1 > \lambda_2 > \cdots > \lambda_n$ and $l_1 \geq l_2 \geq \cdots \geq l_n$. We deduce that

$$k^{1/2}(l_i - \lambda_i)/\lambda_i \sim N(0,1). \tag{11}$$

If z_α is the one-tailed $100(1 - \alpha)\%$ upper significance point of the $N(0,1)$ distribution, then (11) implies the following (approximate) $100(1 - \alpha)\%$ one-sided confidence bound for λ_n:

$$\lambda_n \leq l_n / \left(1 - z_\alpha / k^{1/2}\right) \sim l_n + l_n z_\alpha / k^{1/2}. \tag{12}$$

Similarly, a $100(1 - \alpha)\%$ upper confidence bound for the sum of the $m' = m + 1$ smallest latent roots of Σ is

$$\sum_{j=n-m}^{n} \lambda_j < \sum_{j=n-m}^{n} l_j + \left(\sum_{j=n-m}^{n} l_j^2 \right)^{1/2} z_\alpha / k^{1/2}. \tag{13}$$

Anderson (1963) suggested the following test. If the upper bound in (12) is sufficiently small, then the smallest root λ_n may be taken to be negligible. In the same way, if the upper bound in (13) is sufficiently small, then the m' smallest latent roots of Σ may be deemed negligible.

One advantage of this criterion is that it is based on the implied null hypothesis of the form

$$H_0: \quad \lambda_1 > \lambda_2 > \cdots > \lambda_n > 0.$$

Under this null it is not necessary to be concerned about multiple roots and, in particular, multiple roots of zero. The upper bound (13) is calculated under H and the decision is to reject H if (13) is less than some preassigned small quantity, the idea being that the alternative is much more likely in this event. Just as in the Neyman–Pearson framework, which involves the setting of an arbitrary significance value, the above procedure relies on an arbitrary selection of a 'critical value' against which the upper bounds (12) and (13) are assessed.

Inferential procedures based on the upper bounds (12) and (13) may be used in the present context of tests for cointegration. In this case the null represents

absence of cointegration (Σ nonsingular) and the alternative hypothesis cointegration (Σ singular). However, the latent roots of Σ and S_{Tk} depend on the units of measurement and this presents difficulty in the selection of a 'sufficiently small' criterion for the upper bounds (12) and (13). Anderson (1963) suggests an alternative procedure based on the ratio of the smallest latent roots to the sum of all the latent roots. In this case we obtain by simple manipulations the following $100(1 - \alpha)\%$ upper confidence bound:

$$\frac{\sum\limits_{j=n-m}^{n} \lambda_j}{\sum\limits_{j=1}^{n} \lambda_j} \leq \frac{\sum\limits_{j=n-m}^{n} l_j}{\sum\limits_{j=1}^{n} l_j} + z_\alpha B/k^{1/2}, \tag{14}$$

where

$$B = \left[\left(\sum_{j=n-m}^{n} l_j \right)^2 \left(\sum_{j=1}^{n-m-1} l_j^2 \right) \right.$$

$$\left. + \left(\sum_{j=1}^{n-m-1} l_j \right)^2 \left(\sum_{j=n-m}^{n} l_j^2 \right) \right]^{1/2} / \left(\sum_{j=1}^{n} l_j \right)^2$$

[see Anderson (1984, p. 475)]. Once again, if the upper bound given by (14) is sufficiently small, then the smallest latent roots $\lambda_n, \ldots, \lambda_{n-m}$ may be deemed negligible relative to the sum of all the roots.

It is somewhat easier to assess when the upper bound of (14) is small. Take for example, the case where $m' = 1$ and our focus of attention is the smallest root of Σ. If the upper bound given in (14) is less than $0.10/n$, then we can say with (approximate) $100(1 - \alpha)\%$ confidence that the smallest latent root of Σ is less than 10% of the average of the roots, i.e., $\sum_1^n \lambda_j/n$. This might be interpreted as strong evidence in favor of cointegration (our alternative hypothesis).

The selection of this preassigned small quantity or critical value against which the upper bound (14) is assessed is, of course, quite arbitrary. However, given the inherent arbitrariness in Neyman–Pearson testing, this aspect of the procedure may not be too unpalatable. Note that from the theoretical standpoint the use of such a preassigned quantity has two major effects. First, it ensures that the bounds procedure produces a consistent test. This is because S_{Tk} is consistent for Σ and under the alternative, where Σ has some zero latent roots, the smallest latent roots of S_{Tk} will converge to zero, ensuring that power goes to unity asymptotically. Second, it inevitably leads to a test whose type I error may be large for models in which Σ has one or more small but positive latent roots. The size distortions in the tests that are suffered for

these structures may be tolerated for two reasons. By setting the preassigned quantity to be small the probability of a type I error will be small for all structures except those which have some small latent roots, so that in general the test is likely to be conservative. Moreover, structures for which Σ has some small latent roots are likely to be very difficult to distinguish from cointegrated structures involving economic time series of typical length.

Note that we can apply the test in the opposite direction to provide confirmation of the null hypothesis of no cointegration. Thus, a lower $100(1 - \alpha)\%$ confidence bound for the ratio of the roots is given by

$$\frac{\sum\limits_{j=n-m}^{n} l_j}{\sum\limits_{j=1}^{n} l_j} - z_\alpha B/k^{1/2} \le \frac{\sum\limits_{j=n-m}^{n} \lambda_j}{\sum\limits_{j=1}^{n} \lambda_j}. \tag{15}$$

In this case, if the lower bound (15) is greater than $0.10/n$, then we have $100(1 - \alpha)\%$ confidence that the smallest root of Σ is greater than 10% of the average of the roots, $\sum_1^n \lambda_j/n$. This might be interpreted as substantial support for the absence of cointegration.

The bounds tests based on (14) and (15) relate different latent roots of Σ. If the units of measurement of the variables that comprise y_t in (1) or (1') are all the same this procedure seems justified. However, we may often be interested in situations where the component variables involve different units of measurement. This is most likely to be the case when the long-run equilibrium relationship of interest relates real and monetary aggregates (as in the quantity theory of money). In such situations it seems preferable to work with dimensionless quantities. These may be constructed as follows.

First we define the variance of the innovation sequence $\{u_t\}$ in (1) or (1'), viz.

$$\Sigma_0 = E(u_t u_t').$$

We may require Σ_0 to be nonsingular. Otherwise the support of the distribution of u_t has dimension less than n and there is an exact linear dependence in the series which could be removed prior to the analysis. Now define the matrix

$$P = \Sigma_0^{-1/2} \Sigma \Sigma_0^{-1/2} = I + \sum_{m=1}^{\infty} (\Gamma_m + \Gamma_m'), \tag{16}$$

where

$$\Gamma_m = \Sigma_0^{-1/2} E(u_0 u_m') \Sigma_0^{-1/2},$$

and $\Sigma_0^{1/2}$ is the positive definite square root of Σ_0. Next, we introduce the

sample estimate of Σ_0,

$$S = T^{-1}\Sigma_1^T \hat{u}_t \hat{u}_t',$$

where $\hat{u}_t = \Delta y_t - \overline{\Delta y}$ = measured first differences about the mean. The corresponding sample estimate of P is

$$R = S^{-1/2}S_{Tk}S^{-1/2}. \tag{17}$$

Since $S = \Sigma_0 + O_p(T^{-1/2})$ we now find in place of (10):

$$R \rightarrow \frac{1}{2k} W_n(2k, P),$$

as $T \uparrow \infty$ with k fixed. Define $r_1 \geq r_2 \geq \cdots \geq r_n$ to be the latent roots of R and $\rho_1 > \rho_2 > \cdots > \rho_n$ to be the latent roots of P. In the same way as before, the latent roots r_t are (approximately) independently distributed as $N(\rho_t, \rho_t^2/k)$ for large k. We may therefore deduce confidence bounds for the roots of P and ratios of the roots of P as we did before for Σ.

Thus, for the smallest root ρ_n of P, we have the upper $100(1-\alpha)\%$ confidence limit

$$\rho_n \leq r_n + r_n z_\alpha / k^{1/2}, \tag{18}$$

and the corresponding lower $100(1-\alpha)\%$ confidence limit

$$r_n - r_n z_\alpha / k^{1/2} \leq \rho_n, \tag{19}$$

where, as before, $\Phi(z_\alpha) = 1 - \alpha$ and $\Phi(\cdot)$ is the standard $N(0,1)$ c.d.f. Similarly, upper and lower $100(1-\alpha)\%$ confidence bounds for the ratio of the sum of the m' smallest roots to the sum of all of the roots of P are given by

$$\frac{\displaystyle\sum_{j=n-m}^{n} \rho_j}{\displaystyle\sum_{j=1}^{n} \rho_j} \leq \frac{\displaystyle\sum_{j=n-m}^{n} r_j}{\displaystyle\sum_{j=1}^{n} r_j} + z_\alpha D/k^{1/2}, \tag{20}$$

and

$$\frac{\displaystyle\sum_{j=n-m}^{n} r_j}{\displaystyle\sum_{j=1}^{n} r_j} - z_\alpha D/k^{1/2} \leq \frac{\displaystyle\sum_{j=n-m}^{n} \rho_j}{\displaystyle\sum_{j=1}^{n} \rho_j}, \tag{21}$$

where

$$D = \left(\left(\sum_{j=n-m}^{n} r_j\right)^2 \left(\sum_{j=1}^{n-m-1} r_j^2\right)\right.$$

$$\left. + \left(\sum_{j=1}^{n-m-1} r_j\right)^2 \left(\sum_{j=n-m}^{n} r_j^2\right)\right)^{1/2} \bigg/ \left(\sum_{j=1}^{n} r_j\right)^2.$$

In applying these confidence bounds to tests of cointegration some general guidelines will once again be useful. Since we are now working in terms of dimensionless quantities like correlation coefficients some broadly applicable rules are possible. Thus, for the smallest root bounds given by (18) and (19) we could suggest the following: if the upper bound (18) is less than 0.05, then there is strong evidence in favor of cointegration; if the upper bound is less than 0.01, then the evidence might be taken as being very strong. Conversely, if the lower bound is above 0.05 (respectively, 0.10), then the evidence favors (respectively, strongly favors) the null of no cointegration. For the ratio bounds given by (20) and (21) we might continue to work with the earlier rule based on the value $0.10/n$. For example, if the upper bound (20) is less than $0.10/n$, then the evidence supports the existence of $m' = m + 1$ cointegrating vectors.

Again, we emphasize that these critical values for the bounds are arbitrary. Their use ensures that the bounds tests are consistent but they will lead to size distortion for structures in which Σ has some small latent roots. Such structures will be hard to distinguish from cointegrated structures using these bounds tests.

The adequacy of these broadly defined criteria for the bounds tests has been investigated by simulation methods. The results seem encouraging in terms of both size and power and are reported below in section 5.

4. Tests for invertibility and unit roots

The bounds tests developed in the preceding section may be interpreted as simple tests of the invertibility of the moving average representation of a stationary time series. Note that $\rho_n = 0$ iff $\lambda_n = 0$ (i.e., the smallest latent root of Σ is zero). This is true iff there is a degeneracy in the MA representation of the stationary process u_t. In particular, $\lambda_{\min}(\Sigma) = 0$ iff we can write

$$u_t = D(L)\varepsilon_t = \{AC(L) + (1-L)B(L)\}\varepsilon_t, \tag{22}$$

where A is $n \times n$ of rank $< n$, $C(L)$ and $B(L)$ are matrices of polynomials in the lag operator L, and ε_t is an iid$(0, \Omega)$ sequence of primitive innovations with nonsingular covariance matrix. The degeneracy in $D(L)$ occurs for any

vector γ in the null space of the matrix A. Then $\gamma' D(1) = 0$ and $\gamma' u_t = (1 - L)\gamma' B(L)\varepsilon_t = v_t - v_{t-1}$ for v_t stationary with spectral density $\gamma' B(e^{i\lambda})\Omega B(e^{i\lambda})^* \gamma$. In such cases the MA representation (22) is noninvertible. Test of the invertibility of (22) may therefore be mounted using the bounds test procedures developed in the previous section. Note that the null hypothesis in this case is invertibility and the alternative is noninvertibility (corresponding to Σ and P being of deficient rank). We should remark, in addition, that invertibility may fail due to the existence of a degeneracy in the spectrum $f_{uu}(\lambda) = g(z) = D(z)\Omega D(z)^*$ at a point on the unit circle $z = e^{i\lambda}$ other than $\lambda = 0$. Obviously the bounds tests described earlier are constructed to focus attention on the frequency $\lambda = 0$. Analogous procedures may be developed to explore possible degeneracies at other frequencies.

In the univariate case ($n = 1$) the bounds tests may also be interpreted as tests for the presence of a unit root. We now write $\sigma^2 = \Sigma$, $\sigma_0^2 = \Sigma_0$, $s_{Tk} = S_{Tk}$, $p^2 = P$, and $r^2 = R$. Our interest is in the (alternative) hypothesis

$$H: \quad p^2 = \sigma^2/\sigma_0^2 = 0.$$

We accept H if the upper limit

$$r^2 + r^2 z_\alpha / k^{1/2}$$

is sufficiently small (< 0.05, say) for a preassigned significance level (one-tailed 5%, say). We reject H on the other hand if the lower limit

$$r^2 - r^2 z_\alpha / k^{1/2}$$

is above a preassigned point such as 0.05. Note that when H is true we necessarily have the MA representation

$$u_t = v_t - v_{t-1}$$

[cf. (22) above]. It follows from (1) that $y_t = v_t$ and y_t is stationary. Thus, the alternative hypothesis in this test corresponds to a stationary alternative to (1). Thus, the bounds test based on r^2 here corresponds to a test for the presence of a unit root in the autoregressive representation (1) against a stationary alternative. We remark that this idea has recently been pursued in empirical research by Cochrane (1986) and Campbell and Mankiw (1987). Their approach is to estimate the variance ratio σ^2/σ_0^2 for aggregate time series like GNP and assess its magnitude. Our bounds test formalizes this notion into a statistical test but we work with H as our alternative hypothesis rather than the null.

The limitations of the bounds procedure that were discussed in section 3 also apply here. In particular, the procedure has little discriminatory power between structures with an MA unit root and those with an MA unit root that is close to unity.

5. Monte Carlo results

This section reports the results of a Monte Carlo experiment designed to assess the performance of the latent root bounds tests for no cointegration. We are primarily interested in two issues: (a) the stability of the empirical distribution functions of the bounds statistics to the form of the data generation process (DGP) of the innovation sequence (u_t), and (b) the power of the test procedures under the alternative of cointegration. The simulations were carried out for model sizes ranging from two to five integrated variables and for three different assumptions about the data generation process:

(a) vector MA(1): $u_t = \xi_t + \psi \xi_{t-1}$,

(b) vector AR(1): $u_t = \phi u_{t-1} + \xi_t$,

(c) vector ARMA(1,1): $u_t = \phi^* u_{t-1} + \psi^* \xi_{t-1} + \xi_t$,

where ψ, ϕ, ϕ^*, and ψ^* are the parameter matrices (each $n \times n$) of the process and ε_t is n-vector white noise $(0, I)$.

Estimates of the asymptotic distributions of the various test criteria under the null hypothesis of no cointegration were generated using 500 observations and 5000 repetitions. The innovation sequences for the integrated processes were assumed to be independent so as to ensure that the long-run multiple correlation coefficient between the innovation sequences was zero under the null [see Phillips and Ouliaris (1987, sect. 2)]. Thus, the parameter matrices ψ,

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Notes to the following tables 1 and 2:

(i) The empirical distributions under the null hypothesis of no cointegration were generated using 5000 iterations and 500 observations. Simulations under the alternative hypothesis were based on 2500 iterations, and used sample sizes between 100 and 200 observations

(ii) Model dimension includes the dependent variable.

(iii) The parameters of the vector DGP of $\{u\}_t$ were set to:

Model	MA(1) Diagonal of ψ	AR(1) Diagonal of ϕ	ARMA(1,1) Diagonal of ψ^* and ϕ^*	
1	0.90	0.75	0.89	0.95
2	0.45	0.53	0.77	0.11
3	0.35	0.42	0.12	0.56
4	0.67	0.92	0.45	0 05
5	0.12	0.88	0.34	0.98

(iv) Remaining elements of ψ, ϕ, ψ^*, ϕ^* were set to zero.

(v) Thus, for an ARMA(1,1) model with two integrated variables, the data generation process for the innovation sequence would be as follows:

First process $u_{1t} = 0.89 u_{1,t-1} + 0.95 \varepsilon_{1,t-1} + \varepsilon_{1t}$,
Second process $u_{2t} = 0.77 u_{2,t-1} + 0 11 \varepsilon_{2,t-1} + \varepsilon_{2t}$.

(vi) The random numbers ε_t were drawn from $N(0,1)$ distribution. Random number generator: 'SUPER-DUPER'.

Table 1a

Critical values for bounds tests of no cointegration based on S_{TK} [a]

Model				Size			
	Min	5%	10%	15%	20%	25%	Max
Minimum latent root bound [*inequality (13)*]							
2(a)	1.4503	2.2308	2.3729	2.4729	2.5523	2.6340	4.8233
(b)	2.9681	4.1828	4.4459	4.6194	4.7702	4.8923	9.9191
(c)	9.4610	13.4401	14.3726	15.0415	15.5502	16.0805	31.2651
5(a)	0.8148	1.2406	1.3186	1.3673	1.4111	1.4494	2.4358
(b)	1.8669	2.7031	2.8781	3.0009	3.0998	3.1865	5.6714
(c)	2.0251	2.9079	3.0986	3.2165	3.3194	3.4037	6.1329
Minimum latent bound: Ratio test [*inequality (14)*]							
2(a)	0.4555	0.7358	0.7758	0.8007	0.8225	0.8403	1.2109
(b)	0.4433	0.5709	0.6089	0.6325	0.6541	0.6721	1.1151
(c)	0.1185	0.2226	0.2448	0.2600	0.2731	0.2847	0.7244
5(a)	0.3477	0.4906	0.5190	0.5398	0.5569	0.5717	0.9770
(b)	0.1015	0.1645	0.1779	0.1869	0.1945	0.2016	0.4522
(c)	0.0567	0.1051	0.1140	0.1202	0.1250	0.1294	0.3031

Table 1b

Critical values for bounds tests of no cointegration based on R. [a]

Model				Size			
	Min	5%	10%	15%	20%	25%	Max
Minimum latent bound [*inequality (18)*]							
2(a)	1.3669	1.9253	2.0145	2.0811	2.1312	2.1788	3.3051
(b)	2.5414	3.3412	3.9710	3.5695	3.6469	3.7084	5.3405
(c)	4.4783	5.6640	5.8338	5.9547	6.0406	6.1132	7.5020
5(a)	0.8633	1.2540	1.3259	1.3740	1.4068	1.4410	2.3076
(b)	1.6536	2.4808	2.5937	2.6782	2.7450	2.8064	4.0417
(c)	1.5223	2.2549	2.3617	2.4487	2.5090	2.5519	3.7517
Minimum bound: Ratio test [*inequality (20)*]							
2(a)	0.7584	0.9671	0.9965	1.0165	1.0331	1.0466	1.2151
(b)	0.7688	0.8959	0.9217	0.9354	0.9473	0.9579	1.2090
(c)	0.9489	1.0422	1.0589	1.0693	1.0772	1.0842	1.2136
5(a)	0.5083	0.6804	0.7123	0.7364	0.7564	0.7702	1.1043
(b)	0.4066	0.5891	0.6158	0.6339	0.6426	0.6602	0.9332
(c)	0.4529	0.6084	0.6357	0.6539	0.6673	0.6800	0.9641

Table 2

Rejection rates for bounds tests of no cointegration based on $S_{Tk} = 2\pi \hat{f}_{uu}(0)$ [a]

Model	Obs.	Size					
		Min	5%	10%	15%	20%	25%
Minimum latent bound [*inequality (13)*]							
2(a)	100	84.95	99.20	99.55	99.70	99 75	99 85
(b)		96.05	99.55	99.75	99 90	99.90	99 90
(c)		100.00	100.00	100.00	100.00	100.00	100 00
(a)	150	95.60	99.85	99.85	100.00	100.00	100 00
(b)		92.10	98.60	99 15	99.60	99 75	99 75
(c)		100.00	100.00	100.00	100.00	100 00	100 00
(a)	200	98.15	100.00	100.00	100.00	100.00	100 00
(b)		98.65[*]	100.00	100.00	100.00	100.00	100 00
(c)		100.00	100.00	100.00	100 00	100.00	100 00
5(a)	100	65.30	88.40	90.45	91.55	92 25	92 60
(b)		41.10	83.35	88.05	90.95	92.25	93 90
(c)		66.65	96.45	98.20	98.80	99.20	99 45
(a)	150	76.90	92.40	94.05	94.75	95.20	95 55
(b)		33.80	83.30	88.55	91.80	94 05	95 10
(c)		76.55	98.65	99 25	99 55	99 80	99 90
(a)	200	81.60	94.10	95.40	96.15	96 85	97 00
(b)		26.15	79 35	86.40	90.55	92.35	94 00
(c)		83.65	99.80	100.00	100.00	100.00	100.00
Minimum latent root ratio [*inequality (14)*]							
2(a)	100	80.75	99.30	99.65	99.65	99 75	99 75
(b)		82.05	95.20	96.95	97.40	97 90	98 15
(c)		96.25	99 90	99.95	99 95	99 95	100 00
(a)	150	89.90	99.90	99.95	99 95	99 95	99.95
(b)		92.15	98.60	99.15	99.60	99 75	99 75
(c)		74.55	98.80	99.25	99.50	99 60	99 75
(a)	200	94.25	99.95	100.00	100.00	100 00	100 00
(b)		96.45	99.60	99.85	100 00	100 00	100 00
(c)		82.00	99 50	99.70	99.80	99.90	99 95
5(a)	100	88 35	98.05	98.75	99.32	99.40	99 60
(b)		4.00	26.10	31.90	35.95	39 90	43 55
(c)		9.55	58.05	65.00	69 45	72 80	75 95
(a)	150	91.60	98.70	99.35	99 50	99 85	99 85
(b)		2.75	25 50	32.30	37 80	41.80	45 10
(c)		13.75	72.85	79.35	83.55	86 45	88 65
(a)	200	92.75	99.40	99.65	99 65	99 75	99 75
(b)		1.65	25.95	34 50	40.45	45 35	50 30
(c)		18.35	85.55	90.70	93 15	94 90	95 80

ψ^*, ϕ, and ϕ^* were all set to diagonal matrices. Specific values for the diagonal entries are given in the notes to tables 1 and 2.

Simulations under the alternative of cointegration were conducted for sample sizes equal to 100, 150, and 200 observations, using 2500 iterations. Under the alternative of cointegration the innovation sequences were generated with long-run multiple correlation coefficient, ρ, equal to unity (compared to zero under the null) – again see Phillips and Ouliaris (1987). It was assumed that there was a single cointegrating vector ($m' = 1$) of the form $\gamma' = (1, -i')$, where i is the $(n-1) \times 1$ sum vector and n is (as before) the dimension of the system. Cointegrating vectors of this form appear quite frequently in modern economic models of long-run equilibrium behavior. Examples include the term structure of interest rates, purchasing power parity, and the monetary equation, $MV = PY$. [See Campbell and Shiller (1987) and Engle and Granger (1987) for additional examples.] Finally, the spectral density matrix was estimated by (8), using $k = T^{0.6}$ ordinates of the second-order periodogram, which was computed using the IMSL Fast Fourier transform routine FTRCC.

Tables 1 and 2 and figs. 1 and 2 summarize the results for the critical values and power of the latent root procedures for testing the null hypothesis of no cointegration. These procedures are based on the confidence bounds given by (13), (14), (18), and (20). Note that the critical values of the ratio test are expressed as a proportion of the mean of the latent roots. The simulations show that the critical values of the upper bounds for the minimum latent root [see (13) and (18)] are sensitive to the assumptions made about the form of the DGP (see the first panels of tables 1a and 1b). This, of course, makes it difficult to design simple decision rules for rejecting the null hypothesis of no cointegration. However, the simulations indicate that the probability of obtaining a latent root that is near zero under the null hypothesis of no cointegration is essentially zero. Thus, the minimum upper bound for the smallest latent root is much greater than zero for all the models considered. Moreover, the tabulations reported in table 2 show that the power of the minimum latent root procedure based on S_{Tk} and inequality (13) is high and is seldom below 90% at the 5% level of significance. In the case of the unit free bounds test for the minimum latent root, power was found to be 100%, irrespective of the form of the DGP, the number of observations, and the dimension of the model. This feature of the bounds test is shown clearly in fig. 1 which plots the cumulative distribution function obtained under the null and the alternative for models 5(a)–5(c). As this figure demonstrates, the location of the distributions under the null and the alternative are clearly differentiated and their respective supports are quite disjoint.

In the case of the ratio bounds procedures based on S_{Tk} and inequality (14), we see even stronger evidence of the absence of a zero latent root under the null hypothesis of no cointegration. The minimum upper bound for the smallest latent root as a percentage of the overall mean is less than 10% only
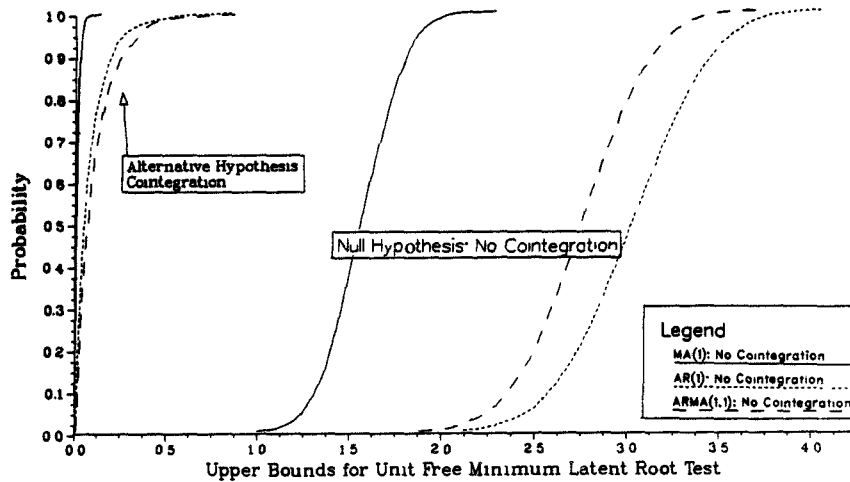
Fig. 1. Cumulative distribution functions for latent root bounds test; unit-free estimator; models 5(a)–(c); eq. (18).
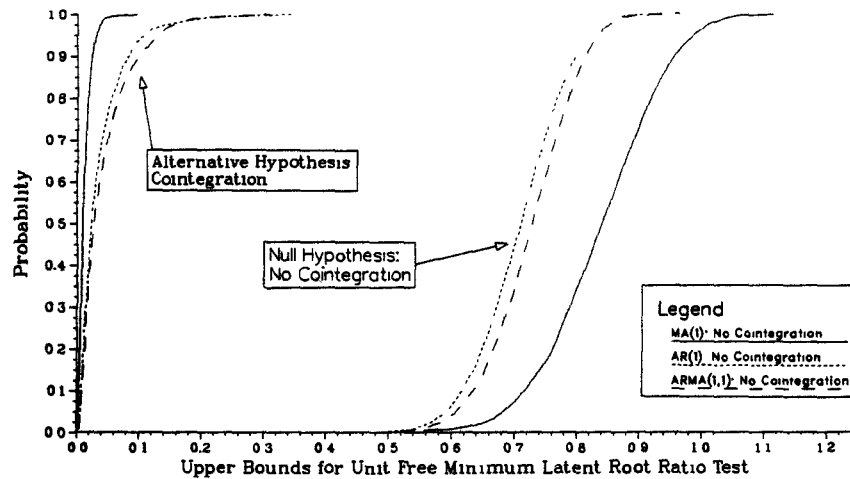

Fig. 2. Cumulative distribution functions for latent root bounds test; unit-free estimator; models 5(a)–(c); eq. (20).

in the case of model 5(c) (see table 1b), and is typically much higher. This, of course, implies that a 10% decision rule for rejecting the null hypothesis would be too conservative, since the true size of the test, at least for the DGP's considered in the tables, would be zero. However, the power of the ratio test, although high for models involving two integrated variables, is low for models 5(b) and 5(c) (see table 2).

Turning to the unit free ratio bounds procedure [based on inequality (20)], we observe that the critical values of the upper bounds are relatively stable across DGPs for the innovation sequence, though they are sensitive to the dimension of the system (see the ratio test panel of table 1b). Power was found to be 100%, once again irrespective of the DGP for the innovation sequence, the number of observations being used. and the dimension of the model. The stability of the upper bounds and the power of the unit free bounds test is shown in fig. 2. Again we see that the distribution of the bounds test under the null and the alternative are clearly differentiated. The figure also highlights the conservative nature of a 10% rejection rule for the unit-free ratio test.

In summary, the Monte Carlo results indicate that latent root procedures are apparently useful tools for detecting the presence or absence of a cointegrating vector. On the basis of these results we feel able to recommend the following diagnostic procedure for testing the null hypothesis of no cointegration:

(1) Evaluate the upper and lower confidence bounds given in (20) and (21) for the ratio of the minimum latent root to the sum of the roots of R.

(2) Reject the null hypothesis of no cointegration if the *upper* bound is less than $0.10/n$.

(3) Accept the null hypothesis of no cointegration if the *lower* bound is greater than $0.10/n$.

The adequacy of this procedure was next evaluated for data whose innovation processes had long-run multiple correlation coefficient in the interval $0 < \rho < 1$ rather than at the extremes ($\rho = 0, 1$).

In what follows we shall restrict our analysis to a simple bivariate system which allows one to control the value of the long-run multiple correlation coefficient. Partition $\{ y_t \}$ as $\{ z_t, x_t \}'$ and consider the following data generation process for the innovation sequence $\{ u_t \}$:

$$u_t = v_t + M v_{t-1},$$

where

$$M = \begin{bmatrix} 0 & 0 \\ b & 0 \end{bmatrix}, \quad b \neq 0,$$

v_t is i.i.d. $N(0, \Sigma_0)$,

and

$$\Sigma_0 = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}.$$

Table 3

Rejection rates for unit-free bounds test [MA(1) model]; rejection rates (%) for different values of a and b (corresponding values of ρ^2 given in parentheses).

a	b						
	0.00	0.50	1.00	1.50	2.50	3 50	5 00
0.25	0.00	0.00	0.00	0.01	8.02	43 26	84 27
	(0.06)	(0.37)	(0.62)	(0.76)	(0.88)	(0 94)	(0 96)
0.50	0.00	0.00	0.00	0 32	3.28	58 25	84 45
	(0.25)	(0.57)	(0.75)	(0.84)	(0.92)	(0 95)	(0 97)
0.75	0.00	0.00	0.06	2.99	52 68	85 83	92 79
	(0.56)	(0.78)	(0.87)	(0.92)	(0 96)	(0 97)	(0 98)

Thus, u_t is driven by an MA(1) process which depends only on a single moving average parameter b. The primary innovations, v_t, are drawn from a multivariate normal distribution with covariance matrix Σ_0 which depends only on the covariance parameter a. The spectral density matrix of u_t evaluated at frequency zero is given by

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ a+b & (a+b)^2 + (1-a^2) \end{bmatrix},$$

from which it follows that [see Phillips and Ouliaris (1987, p. 8)]

$$\rho^2 = \sigma_{12}^2/(\sigma_{22}\sigma_{11}) = \frac{(a+b)^2}{(a+b)^2 + (1-a^2)},$$

and

$$\det(\Sigma) = 1 - a^2.$$

The above model is useful because it allows one to systematically control the degree of cointegration between $\{z_t\}$ and $\{x_t\}$ (as measured by the long-run multiple correlation coefficient). For fixed $|a| < 1$, the degree of cointegration between $\{z_t\}$ and $\{x_t\}$ increases with b. But $\{z_t\}$ and $\{x_t\}$ will never be cointegrated in the sense of Engle and Granger (1987) for any finite value of b when $|a| < 1$ since it is impossible to induce singularity in Σ by varying b in this case. For fixed b, ρ^2 increases and tends to unity as $a \rightarrow \pm 1$. Moreover, when b is finite $\{z_t\}$ and $\{x_t\}$ are cointegrated if and only if $|a| = 1$.

This framework was used to evaluate the performance of the unit-free bounds test using the $0.10/n$ decision rule given above. Table 3 presents the rejection rates (size) of the bounds test for different values of a and b (and hence ρ^2). The empirical distributions were simulated using 5000 replications

and 250 observations. Overall, the results indicate that the unit-free bounds procedure yields a conservative test for models with $\rho^2 < 0.90$, since the rejection rates are well below 5% for all the models in this class. However, the size of the test increases quickly for $\rho^2 > 0.94$ and becomes quite large as ρ^2 approaches 1.00. The rejection rate for some of the models with $\rho^2 > 0.94$ is greater than 80%. For such models, the ability of the bounds procedure to discriminate between the null and the alternative is quite poor.

6. Empirical applications

In this section we apply the unit-free bounds procedure for testing the null hypothesis of no cointegration to five models of potentially cointegrated systems:

(1) total and nondurable consumption and disposable income,
(2) nominal money and income,
(3) the quantity equation $MV = PY$,
(4) real stock prices and dividends, and
(5) the term structure of interest rates.

Models 1–3 were originally formulated as cointegrated systems by Engle and Granger (1987). They found evidence in favor of a cointegrating vector only in the case of real nondurable consumption and disposable income. Their analysis was based on the residuals of the cointegrating regression, using the DW and ADF statistics to detect nonstationarity in this vector. The critical values for the DW and ADF were generated by a small Monte Carlo experiment for an assumed, but arbitrary, DGP for the innovation sequence. Campbell and Shiller (1987) employed these critical values to accept the null hypothesis of no cointegration between real stock prices and dividends, and accept cointegration between short- and long-term yields on bonds (the one-month and twenty-year yields, respectively).

Table 4 presents the results of applying the unit free bounds procedures to the above models. The innovations for the system were estimated by the measured differences of the original series. Using the 10% decision rule for the ratio test, we find that the null hypothesis of no cointegration cannot be rejected for models 3–5. Indeed, there is strong evidence against cointegration. For example, in the case of the real quantity equation, the lower bounds for the minimum latent root, expressed either in absolute terms or as a proportion of the average root, are not close to zero. This result holds true for all definitions of the money supply – M1, M2, M3, and total liquid asset holdings. The results for consumption and disposable income (model 1) are inconclusive, since the lower bound of the minimum latent root as a ratio of

Table 4

Empirical results for cointegration models; unit-free estimator.[a]

Model	T	k	Bounds					
			r_n	$(r_n/\bar{r})$	r_n^U	r_n^L	$(r_n/\bar{r})^U$	$(r_n/\bar{r})^L$
1. Consumption and disposable income; real, per capita, 1982$; 1947:2–1986:1								
(a) Nondurable	156	12	0.3120	0.1508	0.7187	0.1903	0.5066	0 0966
(b) Total			0.2814	0.1271	0.6482	0.1716	0.4319	0.0767
2. Nominal money and income; 1959:3–1986:1								
(a) M1	108	10	0.4504	0.1925	1.1846	0.2655	0.6567	0.1122
(b) M2			0.3663	0.1613	0.9634	0.2158	0.5598	0.0854
(c) M3			0.3921	0.1740	1.0314	0.2310	0.6000	0 0960
(d) Liquid assets			0.3795	0.2119	0.9981	0.2234	0.7166	0.1310
3. Quantity equation $MV = PY$; 1959:3–1986:1								
(a) M1	108	10	1.8257	0.2077	4.8019	1.0762	1.0092	0 2379
(b) M2			1.6355	0.1690	4.3009	0.9641	0.8299	0.1843
(c) M3			1.6027	0.1552	4.2154	0.9447	0.7657	0.1656
(d) Liquid assets			1.3769	0.1351	3.6217	0.8116	0 6724	0.1378
4. Stock prices and dividends; 1872–1985								
	114	10	0.5556	0.2881	1.4561	0.3262	0.9359	0.2167
5. Term structure (one-month, twenty-year yield); 1959:4–1983:1								
	296	17	0.4010	0.2894	0.7644	0.2595	0.8555	0.3024

[a]Data for models 1–3 are from the June edition of the 1986 Citibase databank. Data for models 4 and 5 were kindly provided by J.Y. Campbell and R. Shiller.

the overall mean is less than 10%. In the case of model 2, which tests the long-run relationship between nominal income and money, the bounds test is also inconclusive using M2 and M3.

Some observations on these results are in order. First, acceptance of no cointegration for real stock prices and dividends is consistent with the results of Shiller (1981), who rejected the present value model of stock prices using volatility bounds tests. The results for the term structure of interest rates are consistent with previous results for the rational expectations theory of the term structure [see Shiller (1986, table 2) for a summary of these results]. Given the observed empirical regularity of the relationship between consumption and income, the inconclusive outcome of the bounds test for model 1 is most likely a result of the 10% bounds rule. Finally, accepting the null hypothesis of no cointegration between narrowly defined measures of money (M1) and income is consistent with the empirical observation that the velocity of money, for narrowly defined definitions of the money supply, has behaved erratically since the deregulation of the banking system in 1981.

7. Conclusion

Testing for the presence of cointegration amongst aggregate economic time series seems likely to become a standard method of assessing the empirical support for steady state theories of macroeconomic behavior. Cointegrated systems capture the idea that individual economic time series often exhibit nonstationary characteristics but that certain combinations of the series move together over time. Since linear combinations of nonstationary variables are typically also nonstationary, evidence that a cointegrating vector exists obviously provides strong support for the existence of a long-run relationship amongst a group of integrated variables.

This paper develops diagnostic procedures that are designed to help detect the presence of cointegration in multiple time series. Drawing on earlier work, we provide necessary and sufficient conditions for cointegration. These conditions prescribe the behavior of $f_{uu}(\lambda)$, the spectral density matrix of the innovations, in a neighborhood of the origin. Under cointegration, $\Sigma = 2\pi f_{uu}(0)$ is singular and its smallest latent root is zero. These results motivate a class of statistical tests for cointegration that are based on principal components methods. The tests place upper and lower bounds on the minimum latent root (and ratio of the minimum to the average latent root) of a unit-free form of the matrix. The resulting bounds tests are simple to construct, involve the standard normal distribution and yield criteria for acceptance and rejection of cointegration. Monte Carlo simulations indicate that the bounds test provides a useful diagnostic procedure for testing the null hypothesis of no cointegration.

We emphasize that our diagnostic procedures do not purport to test a null hypothesis of cointegration or, more specifically, $\lambda_{min}(\Sigma) = 0$. There are pitfalls in classical Neyman–Pearson tests of the null $\lambda_{min}(\Sigma) = 0$ and these have recently been considered in Phillips and Ouliaris (1987). In effect, no generally applicable asymptotic theory is possible under this null and classical tests turn out to be inconsistent. Our approach in the present paper works from a null hypothesis of no cointegration. This enables us to use conventional asymptotic methods to assess whether any latent roots of Σ are negligible. The methods we recommend do have a precedent in principal components theory. Indeed, they may be regarded as performing a form of principal components analysis for time series.

The unit-free bounds procedure recommended in section 3 yields a test which is consistent against the alternative hypothesis of a cointegrated system. This is achieved by the use of a preassigned fixed cut-off point [we suggested that the upper bound (20) be less than $0.10/n$] to identify cointegrated systems. For some time series, however, the long-run multiple correlation coefficient may be large but not unity. In such cases the bounds procedure can suffer size distortions whereby the probability of rejection of the null by the

bounds criterion is high even though the series are not cointegrated. This is borne out by some of our simulation results for $\rho^2 > 0.90$. For such series the discriminatory power of the bounds test between the null and the alternative is low. This is a limitation of the bounds test. Interestingly, it does not appear to be a serious problem in the case of the empirical implementations reported in section 5. For in these applications the evidence is predominantly in favor of the null of no cointegration.

References

Anderson, T.W., 1963, Asymptotic theory for principal component analysis, Annals of Mathematical Statistics 34, 122–148.

Anderson, T.W., 1984, An introduction to multivariate statistical analysis, 2nd ed (Wiley, New York).

Bhargava, A., 1986, On the theory of testing for unit roots in observed time series. Review of Economics Studies 53, 369–384.

Brillinger, D.R., 1981, Time series: Data analysis and theory (Holden-Day, San Francisco, CA)

Campbell, J.Y., 1986, Does saving anticipate declining labor income? An alternative test of the permanent income hypothesis, Econometrica, forthcoming.

Campbell, J.Y. and N.G. Mankiw, 1987, Permanent and transitory components in macroeconomic fluctuations, AEA Papers and Proceedings 77, 111–117.

Campbell, J.Y and R.J. Shiller, 1987, Cointegration and tests of present value models. Journal of Political Economy 97, 1062–1088.

Cochrane, J.H., 1986, How big is the random walk in GNP, Mimeo (Chicago University, Chicago, IL).

Corbae. P.D. and S. Ouliaris, 1987, Cointegration and tests of purchasing power parity, Review of Economics and Statistics, forthcoming

Engle, R.F. and C.W.J. Granger, 1987, Cointegration and error correction: Representation, estimation and testing, Econometrica 55, 251–276.

Granger, C.W.J.. 1981. Some properties of time series data and their use in econometric model specification, Journal of Econometrics 28, 121–130.

Granger, C.W.J., 1983, Co-integrated variables and error correcting models, Unpublished discussion paper no. 83-13a (University of California, San Diego, CA).

Granger, C.W.J., 1986, Developments in the study of cointegrated economic variables, Oxford Bulletin of Economics and Statistics 48, 213–228.

Granger, C.W.J. and A.A. Weiss, 1983, Time series analysis of error correction models, in: S Karlin, T. Amemiya and L.A. Goodman. eds.. Studies in econometric time-series and multivariate statistics (Academic Press, New York).

Hendry, D.F., 1986, Econometric modelling with cointegrated variables: An overview, Oxford Bulletin of Economics and Statistics 48, 201–212.

Muirhead, R.J., 1982, Aspects of multivariate statistical theory (Wiley, New York).

Park, J.Y. and P.C.B. Phillips, 1986, Statistical inference in regressions with integrated processes: Part 1, Cowles Foundation discussion paper no 811 (Yale University, New Haven, CT)

Park, J.Y and P.C.B. Phillips, 1987, Statistical inference in regressions with integrated processes. Part 2, Cowles Foundation discussion paper no 819 (Yale University, New Haven, CT)

Phillips, P.C.B., 1986a, Understanding spurious regressions in econometrics. Journal of Econometrics 33, 311–340.

Phillips, P C.B., 1986b. Weak convergence to the matrix stochastic integral $\int_0^1 B\,dB'$, Journal of Multivariate Analysis, forthcoming.

Phillips, P.C.B., 1987a, Time series regression with a unit root, Econometrica 55, 277–301

Phillips. P C.B.. 1987b, Partially identified econometric models. Cowles Foundation discussion paper no. 845 (Yale University, New Haven, CT)

Phillips, P.C.B. and S.N Durlauf, 1986, Multiple time series with integrated variables, Review of Economic Studies 53, 473–496.

Phillips, P.C.B., and S. Ouliaris, 1987, Asymptotic properties of residual based tests for cointegration, Cowles Foundation discussion paper no. 847 (Yale University, New Haven, CT).

Priestley, M.B., 1981, Spectral analysis and time series (Academic Press, London).

Said, S.E. and D.A. Dickey. 1984, Testing for unit roots in autoregressive-moving average models of unknown order, Biometrika 71, 599–607.

Sims, C.A., 1978, Least squares estimation of autoregressions with some unit roots, CERDE discussion paper no. 78-95 (University of Minnesota, Minneapolis, MN).

Sims, C.A., 1986, Asymptotic normality of coefficients in a vector autoregression with unit roots, Presented at the summer Econometric Society meetings (Duke University, Durham, NC).

Shiller, R.J., 1981, Do stock prices move too much to be justified by subsequent changes in dividends?, American Economic Review 71, 421–436.

Shiller, R.J., 1986, The term structure of interest rates, forthcoming in: Handbook of monetary economics (North-Holland, Amsterdam).

Stock, J.H. and M.W. Watson, 1986, Testing for common trends, Institute of Economic Research discussion paper no. 1222 (Harvard University, Cambridge, MA).