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DOES GNP HAVE A UNIT ROOT?
A re-evaluation

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Stock and Watson (1986) test the hypothesis that real per capita GNP has a unit root by using a test statistic due to Phillips (1985) which incorporates a non-parametric correction for the serial correlation induced by system and error dynamics The version of this test that is used by Stock and Watson does not accommodate the presence of a drift, and to compensate, they detrend the series by extracting a 1 5% annual trend growth We use a version of this class of non-parametric tests, developed by Phillips and Perron (1986), which allows for an estimated drift, and reassess the Stock and Watson findings

1. Introduction

Many historical macroeconomic time series display a strong pattern of secular growth over time. A problem of some importance to both theorists and empirical researchers is whether this observed non-stationarity is stochastic or deterministic If a particular series has a unit root (with possibly a non-zero drift) then the trend is (in part) stochastic and involves the accumulation of random innovations, each of which has an enduring effect on the future trajectory of the time series. On the other hand, if the series has no root on the unit circle and the non-stationarity is well represented by stationary fluctuations about a deterministic trend, then the random innovations have only a temporary influence on the historical trajectory of the series It is an empirical question which of these alternatives is the more realistic.

Recently Nelson and Plosser (1982) used the Dickey–Fuller (1979) test procedure to distinguish between these two hypotheses for a selected group of major U.S. macroeconomic time series. They found that all but one of the series tested have a unit root (the exception being unemployment) and they concluded that the non-stationarity of macro time series is in general stochastic. The Dickey–Fuller test for a unit root uses an autoregressive (parametric) correction to account for the short-run dynamics of the process. If the first-difference representation of a series contains an important moving average component this implies that a potentially large number of nuisance parameters must be estimated *and* that this number should be allowed to grow with the sample size at a controlled rate in order to validate the asymptotic theory [see Said and Dickey (1984)]. Since two degrees of freedom are lost for each additional parameter estimated, [1] this procedure seems likely to have low power in such cases, at least in finite samples.

Theoretical and empirical considerations support the view that moving average terms are present in many macroeconomic time series [Schwert (1985) provides a recent discussion of some of the evidence]. Accordingly, it seems desirable to have an alternative testing procedure which does not rely on an autoregressive correction to account for the short-run dynamics. Such a test has been developed by Phillips (1985). The new test involves a non-parametric correction for the serial correlation that is induced by short-run dynamics (in both the system and the errors) and it has rather a wide applicability in the present context. Recently, Stock and Watson (1986) analysed the real per capita GNP series using Phillips' test. However, the form of the test that they employ does not allow for a fitted drift. To circumvent this problem Stock and Watson detrended the data by extracting a 1.5% annual trend growth so as to induce a driftless series.

In fact, the test procedures in Phillips (1985) have recently been extended by Phillips and Perron (1986) and Perron (1986a,b) to accommodate a fitted time trend and a non-zero drift. The resulting tests may be used directly to address the issue of whether GNP has a unit root. The purpose of this letter is to report the results of these tests.

Our letter is organized as follows. In section 2 we argue that the procedure used by Stock and Watson may yield misleading results. Section 3 presents the non-parametric tests valid in the presence of drift and section 4 applies these tests to the various GNP series studied by Stock and Watson. The conclusions are summarized in section 5.

2. Problems caused by the presence of a drift

The problems caused by the presence of a drift for the estimator of the serial correlation coefficient are most easily analysed by considering the asymptotic distribution of the usual estimators. Let $\hat{\alpha}$ be the coefficient estimate in a regression of y_t against y_{t-1} and $t_{\hat{\alpha}}$ be its t-statistic. Similarly, let α^* and t_{α^*} be the corresponding statistics in a regression that includes a constant. Then, it is easily shown [2] [see Perron (1986a)] that, if there is a non-zero drift (μ) in the series, as $T \uparrow \infty$:

(a) $T^{1/2}(T(\hat{\alpha}-1)-3/2) \to \mathrm{N}(0, 9\sigma^2/5\mu^2)$,

(b) $T^{-1/2}t_{\hat{\alpha}} \underset{p}{\to} \sqrt{3/2}\left(\sigma_u^2+\mu^2/4\right)^{1/2}$,

(c) $T^{3/2}(\alpha^*-1) \to \mathrm{N}(0, 12\sigma^2/\mu^2)$,

(d) $t_{\alpha^*} \to \mathrm{N}(0, \sigma^2/\sigma_u^2)$,

where $\sigma^2 = \lim_{T\to\infty} T^{-1}\mathrm{E}(S_T^2)$, $S_T = \Sigma_{t=1}^T u_t$, $\sigma_u^2 = \lim_{T\to\infty} T^{-1}\Sigma_{t=1}^T \mathrm{E}(u_t^2)$, and $\{u_t\}$ are the residuals defined in the following model:

$$y_t = \mu + y_{t-1} + u_t. \tag{1}$$

The rate of convergence of the estimators to their true value is of course much faster with a

[1] One degree of freedom is lost because of the extra parameter estimated and one because of the need for an extra initial observation.

[2] Some conditions on the innovation sequence $\{\mu_t\}$ are assumed. See Phillips (1985) and Phillips and Perron (1986).

non-zero drift Furthermore, the limiting distribution depends upon the drift parameter μ. This parameter can be consistently estimated by the fitted constant in the regression model (1). However, in finite samples, this estimator is not invariant to the initial observation y_0 and a test based upon it would depend upon the units of measurements This solution is not appealing.

The procedure followed by Stock and Watson (1986) is to assume a known fixed drift of 1.5% annual trend growth [i.e., $\mu = 1.5$ in model (1) with y_t representing the logarithm of real per capita GNP] This procedure is valid only insofar as the true drift is in fact 1.5%. There is, of course, no way of knowing this in practice. Indeed, some growth in the series is consistent with a driftless random walk and the effects of inappropriate detrending in this and other cases have been well documented by Nelson and Kang (1981) and Durlauf and Phillips (1986). The simplest way of dealing with this problem is to employ a version of the Phillips test which accommodates a non-zero drift.

3. The test statistics

Let $\{y_t\}$ be a series generated according to model (1) with the innovation sequence satisfying some appropriate moment and mixing conditions [3] Consider the following regression equation which is to be estimated by least squares:

$$y_t = \mu + \beta(t - T/2) + \alpha y_{t-1} + u_t. \tag{2}$$

The null hypothesis of interest is that $\alpha = 1$ and $\beta = 0$ within a maintained hypothesis which permits a possibly non-zero drift μ Three tests are possible. The first uses the standardized and centered least squares estimates of α, viz $T(\tilde{\alpha} - 1)$. The other statistics are: the t-statistic on α, $t_{\tilde{\alpha}}$ (for $\alpha = 1$), and the regression 'F-test' Φ_3, studied by Dickey and Fuller (1981) for the special case of iid innovation. Φ_3 is defined as

$$\Phi_3 = (2\tilde{s})^{-1}\left[T\left\{s_0^2 - \left(\bar{y}_{(0)} - \bar{y}_{(-1)}\right)^2\right\} - (T-3)\tilde{s}^2\right], \quad \text{with}$$

$$\tilde{s}^2 = (T-3)^{-1}\sum_{t=1}^{T}\left(y_t - \tilde{\mu} - \tilde{\beta}(t - T/2) - \tilde{\alpha} y_{t-1}\right)^2,$$

$$s_0^2 = T^{-1}\sum_{t=1}^{T}\left(y_t - y_{t-1}\right)^2, \qquad \bar{y}_{(-i)} = T^{-1}\sum_{t=1}^{T} y_{t-i} \qquad (i = 0, 1).$$

The new statistics proposed by Phillips and Perron (1986) and Perron (1986b) which permit a wide class of innovation sequences $\{u_t\}$ and which are invariant with respect to the drift parameter μ in (2) are

$$Z(\tilde{\alpha}) = T(\tilde{\alpha} - 1) - \left(T^6/24D_x\right)\left(\tilde{\sigma}_{Tl}^2 - \tilde{s}^2\right), \tag{3}$$

$$Z(t_{\tilde{\alpha}}) = (\tilde{s}/\tilde{\sigma}_{Tl})t_{\tilde{\alpha}} - \left(T^3/4\sqrt{3}\,D_x^{1/2}\tilde{\sigma}_{Tl}\right)\left(\tilde{\sigma}_{Tl}^2 - \tilde{s}^2\right), \tag{4}$$

$$Z(\Phi_3) = (\tilde{s}/\tilde{\sigma}_{Tl})\Phi_3 - \left(1/2\tilde{\sigma}_{Tl}^2\right)\left(\tilde{\sigma}_{Tl}^2 - \tilde{s}^2\right)\left[T(\tilde{\alpha} - 1) - \left(T^6/48D_x\right)\left(\tilde{\sigma}_{Tl}^2 - \tilde{s}^2\right)\right] \quad \text{where,} \tag{5}$$

[3] These conditions are general enough to include all finite ARMA processes generated by Gaussian errors [see Phillips and Perron (1986, assumption 2 1)]

$$D_X = \left(T^2(T^2-1)/12\right)\Sigma y_{t-1}^2 - T(\Sigma t y_{t-1})^2 + T(T+1)\Sigma t y_{t-1}\Sigma y_{t-1} - \left(T(T+1)(2T+1)/6\right)(\Sigma y_{t-1})^2$$

is the determinant of the matrix $(X'X)$ where X is the matrix of regressors defined by regression model (2). $\tilde{\sigma}_{Tl}^2$ is an estimator of $\sigma^2 = \lim_{T\to\infty} T^{-1}\mathrm{E}(S_T^2)$ and is the non-parametric counterpart of the autoregressive correction in the Dickey–Fuller procedure. Various consistent estimators are possible [see Phillips (1985) and Perron (1986c) for more details and discussion]. The one adopted here was proposed by Newey and West (1985). It is defined by

$$\tilde{\sigma}_{Tl}^2 = T^{-1}\sum_{1}^{T}\tilde{u}_t^2 + 2T^{-1}\sum_{\tau=1}^{l} w_{\tau l}\sum_{t=\tau+1}^{T}\tilde{u}_t\tilde{u}_{t-\tau}$$

where $w_{\tau l} = 1-\tau/(l+1)$ and the $\tilde{u}_t$'s are the regression residuals. As in the Dickey–Fuller case, there is some ambiguity in the appropriate choice of the lag truncation number l although theoretical results provide some general guidelines [Phillips (1985)]. We shall present our empirical results for a range of values of l so that any sensitivities in the outcome of the tests are apparent.

A major advantage of the statistics (3)–(5) is that they have the same asymptotic distributions for a very wide class of error structures, as the simple statistics developed by Dickey and Fuller (1979, 1981) have under the assumption of iid errors. The critical values of our statistics (3)–(5) are therefore the same and may be found in Fuller (1976) and Dickey and Fuller (1981). Moreover, as shown in Phillips (1986) and Phillips and Perron (1986) there is no loss in asymptotic power against a sequence of local alternatives in the use of these test statistics

4. Empirical results

We use the same real per capita GNP series in our empirical work as those used by Stock and Watson (1986): (1) the Nelson–Plosser series (annual from 1909 to 1970); (2) the Friedman–Schwartz

Table 1

Series/period	$Z(\tilde{\alpha})$ test statistics [a]						
	$\tilde{\alpha}$	$l=0$	$l=1$	$l=3$	$l=6$	$l=9$	$l=12$
Nelson–Plosser							
1909–1970	0.868	−8.18	−10.46	−11.58	−9.74	−8.52	−6.47
1909–1940	0.751	−7.97	−10.12	−10.51	−8.11	−4.93	−3.00
1941–1970	0.730	−8.10	−10.65	−11.99	−9.24	−7.70	−7.00
1946–1970	0.676	−8.10	−8.11	−9.43	−8.63	−5.97	−3.88
Friedman and Schwartz							
1869–1940	0.760	−17.28	−19.36	−19.65	−17.02	−14.05	−11.96
1869–1908	0.651	−13.96	−13.99	−13.45	−13.16	−11.33	−9.33
1869–1919	0.582	−21.32	−20.76	−20.79	−20.58	−18.47	−15.75
1909–1940	0.798	−6.46	−8.65	−9.18	−7.38	−5.46	−3.62
1869–1975	0.820	−19.26	−22.60	−23.90	−21.09	−19.46	−16.34
1941–1975	0.741	−9.07	−11.46	−12.47	−9.70	−7.85	−7.02
1946–1975	0.720	−8.40	−9.15	−9.36	−8.97	−6.20	−3.24
NIPA							
1947 I–1986 I (quarterly)	0.956	−6.91	−9.45	−12.54	−12.77	−12.09	−11.41
1947–1984 (annual)	0.725	−10.45	−12.01	−11.40	−10.15	−8.28	−6.21

[a] Critical values: 10% −18.3, 5% −21.8, 2.5% −25.1, 1% −29.5 [Source: Fuller (1976)]

Table 2

Series/period	$Z(t_{\tilde{\alpha}})$ test statistics [a]						
	$\tilde{\alpha}$	$l=0$	$l=1$	$l=3$	$l=6$	$l=9$	$l=12$
Nelson – Plosser							
1909–1970	0 868	−2 18	−2 43	−2 55	−2 36	−2 33	−2 00
1909–1940	0 751	−1 95	−2 24	−2 28	−2 00	−1 55	−1 20
1941–1970	0 730	−2 17	−2 47	−2 60	−2 32	−2 16	−2 08
1946–1970	0 676	−2 24	−2 28	−2 41	−2 33	−2 06	−1 89
Friedman and Schwartz							
1869–1940	0 760	−3 15	−3 32	−3 34	−3 14	−2 91	−2 73
1869–1908	0 651	−2 82	−2 86	−2 81	−2 78	−2 62	−2 43
1869–1919	0 582	−3 62	−3 61	−3 62	−3 60	−3 46	−3 29
1909–1940	0 798	−1 66	−1 98	−2 05	−1 82	−1 53	−1 21
1869–1975	0 820	−3 24	−3 50	−3 59	−3 39	−3 26	−3 02
1941–1975	0 741	−2 25	−2 53	−2 62	−2 35	−2 15	−2 05
1946–1975	0 720	−2 00	−2 12	−2 15	−2 10	−1 74	−1 25
NIPA							
1947 I–1986 I (quarterly)	0 956	−1 86	−2 18	−2 51	−2 53	−2 46	−2 39
1947–1984 (annual)	0 725	−2 44	−2 61	−2 55	−2 43	−2 24	−2.01

[a] Critical values 10% −3 12, 5% −3 41, 2,5% −3 66, 1% −3 96 [Source Fuller (1976)]

Table 3

Series/period	$Z(\Phi_3)$ test statistics [a]					
	$l=0$	$l=1$	$l=3$	$l=6$	$l=9$	$l=12$
Nelson – Plosser						
1909–1970	2 55	3 10	3 36	2 93	2 66	2 24
1909–1940	1 88	2 48	2 58	1 98	1 17	0 66
1941–1970	2 13	2 88	3 23	2 51	2.09	1 89
1946–1970	2 34	2 42	2 76	2 55	1 86	1 28
Friedman and Schwartz						
1869–1940	4 75	5 35	5 42	4 75	3 98	3.43
1869–1908	3 81	3 90	3 76	3 69	3 21	2 68
1869–1919	6 17	6 13	6 14	6 09	5 54	4 83
1909–1940	1.46	2.03	2 16	1 72	1 28	0 87
1869–1975	5 10	5 98	6 31	5 60	5 18	4 38
1941–1975	2 38	3 08	3 33	2 62	2 13	1 90
1946–1975	1 99	2 24	2 29	2 20	1 50	0 75
NIPA						
1947 I–1986 I (quarterly)	1 90	2 50	3 24	3 30	3 14	2 97
1947–1984 (annual)	2 88	3 34	3 19	2 87	2 40	1 87

[a] Critical values 10% 5 34, 5% 6 26, 2,5% 7 16, 1% 8 27 [Source Dickey and Fuller (1981)]

(1982) series (annual from 1869 to 1975), (3) the National Income and Product Accounts (annual from 1947 to 1984 and quarterly from 1947 I to 1986.I). The results are presented in tables 1 through 3 for each test statistic.

Consider the results for the Nelson–Plosser series first. In all cases (covering the statistics used and the sub-samples considered) we cannot reject the null hypothesis of a unit root without trend (at conventional significance levels). We can reject the same null hypothesis at close to the 5% level using the Friedman–Schwartz series from 1869 to 1975. When analysing sub-samples of this series we can reject the null hypothesis using the samples 1869–1940 and 1869–1919 (at a significance level between 5 and 10%) but not using other sub-samples It therefore appears from this evidence that the behavior of the series is different before and after World War II. [4] With both the quarterly and annual NIPA series there is no evidence to reject the unit root hypothesis.

5. Conclusions

The conclusions are mixed and depend upon the series used and the sample periods analyzed. Working at conventional levels of significance the Nelson–Plosser series do not permit rejection of the null hypothesis of a unit root both before and after World War II. The Friedman–Schwartz series permit rejection before World War II but not after (though we can reject for the whole sample) The clearest rejection occurs for the period 1869–1919. Finally the NIPA series does not permit rejection for a post-war sub-sample. [5] One possible explanation for these divergences concerns the power of the tests relative to the span of observation. As shown by Shiller and Perron (1985), the power of unit root tests depends on the span of the available data. We reject the null hypothesis for the Friedman–Schwartz series which has the largest span, 107 years By contrast, a maximum of only 40 years of data is available for the post-war period. Here the tests may have low power and the Nelson–Plosser series, while covering both the pre-war and post-war series, contains only 62 observations.

References

Dickey, D A and W A Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427–431

Dickey, D A and W A Fuller, 1981, Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica 49, 1057–1072

Durlauf, S and P C B Phillips, 1986, Trends versus random walks in time series analysis, Cowles Foundation discussion paper no 788 (Yale University, New Haven, CT)

Friedman, M and A J Schwartz, 1982, Monetary trends in the United States and the United Kingdom Their relation to income, prices and interest rates, 1867–1975 (University of Chicago Press, Chicago, IL)

Fuller, W A , 1976, Introduction to statistical time series (Wiley, New York)

Harvey, A C , 1985, Trends and cycles in macroeconomic time series, Journal of Business and Economic Statistics 3, 216–227

Nelson, C R and H Kang, 1981, Spurious periodicity in inappropriately detrended time series, Econometrica 49, 741–751

Nelson, C R and C I Plosser, 1982, Trends and random walks in macroeconomic time series Some evidence and implications, Journal of Monetary Economics 10, 139–162

[4] See Harvey (1985) who noted some similar changes

[5] These conclusions are robust to possible changes in the estimation technique We tried a variety of combinations with the following possible alternatives (a) using the logarithm or the level of the variables, (b) using the residuals under the null or under the alternative hypothesis in the estimation of the variances σ^2_{Tl}, (c) using the Newey–West weighting function $w_{\tau l}$ or constraining it to $w_{\tau l} = 1$ All these different variations yielded roughly the same conclusions

Newey, N K and K D West, 1985, A simple positive definite heteroskedasticity and autocorrelation consistent covariance matrix, Princeton discussion paper no 92 (Princeton University, Princeton NJ)

Perron, Pierre, 1986a, Hypothesis testing in time series regression with a unit root, Unpublished Ph D dissertation (Yale University, New Haven, CT)

Perron, Pierre, 1986b, Tests of joint hypotheses for time series regression with a unit root, Cahier 2086 (Centre de recherche et développement en économique, Université de Montréal, Montreal)

Perron, Pierre, 1986c, Trends and random walks in macroeconomic time series Further evidence from a new approach, Mimeo (Université de Montréal Montreal)

Phillips, P C B , 1985, Time series regression with a unit root, Cowles Foundation discussion paper no 740-R (Yale University, New Haven, CT) to appear in Econometrica, 1986

Phillips, P C B , 1986, Regression theory for near integrated time series, Cowles Foundation discussion paper no 781 (Yale University, New Haven, CT)

Phillips, P C B and P Perron, 1986, Testing for a unit root in time series regression, Cowles Foundation discussion paper no 795 (Yale University, New Haven, CT)

Said, S E and D A Dickey, 1984 Testing for unit roots in autoregressive-moving average models of unknown order, Biometrika 71, 599–608

Schwert, G W , 1985, Effects of model specification on tests for unit roots, Manuscript (University of Rochester, Rochester, NY)

Shiller, Robert J and P Perron, 1985, Testing the random walk hypothesis Power versus frequency of observation, Economics Letters 18, 381–386

Stock, J H and M W Watson, 1986, Does GNP have a unit root?, Economics Letters 22, nos 2–3, 147–151