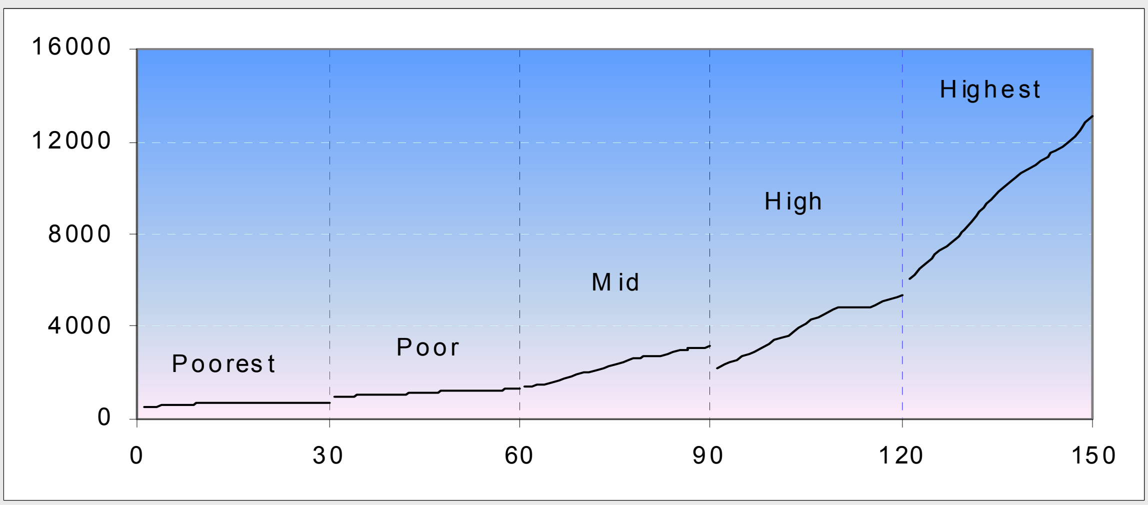
New Methods for Time Series and Panel Econometrics



Average Real per Capita Income over 1960-1989 with Country Groupings

Peter C. B. Phillips

Cowles Foundation, Yale University

IMF Seminar: September 29, 2003

Seminar 2002

- **Limitations of the Econometric Approach**
  - Laws of Econometrics
  - Limits to Empirical Knowledge & Forecasting
  - Proximity Theorems
  - A Look to the Future
  - Online Econometric Services

- **Dynamic Panel Modeling**
- **Estimation of Long Memory**

Outline

- **Dynamic Panels with Incidental Trends & Cross Section Dependence**
  - Bias & Inconsistency
  - Adjusting for Bias
  - Homogeneity testing
  - Modeling & Handling Cross Section Dependence
- **Nonstationary Panel Models**
  - Unit Roots, Near unit roots, incidental trends
  - Testing unit roots & CSD
  - Cointegration & spurious regression
- **Applications**
  - Growth convergence & transitions
  - FH savings/investment regressions
  - Bias corrections – PPP & demand for gas

Papers

- Phillips & Moon (1999). Linear regression limit theory for nonstationary panel data, *Econometrica,* 67, 1057-1111.

- Moon & Phillips (1999). Maximum likelihood estimation in panels with incidental trends. *Oxford Bulletin of Economics and Statistics*, 61,711–48.

- Phillips & Sul (2003). Dynamic panel estimation and homogeneity testing under cross section dependence. *Econometrics Journal*, 6, 217-259.

- Phillips & Sul (2003). Bias in Dynamic Panel Estimation with Fixed Effects, Incidental Trends and Cross Section Dependence. CFDP # 1438, Yale University

- Moon, Perron & Phillips (2003). Incidental trends and the power of unit root tests. CFDP # 1435, Yale University

http://cowles.econ.yale.edu/

Dynamic Panel Models

Latent variable equation

$$y^*_{i,t} = \rho y^*_{i,t-1} + u_{i,t}, \quad u_{i,t} \sim iidN(0, \sigma_i^2)$$

$$\rho \in (-1, 1]$$

Panel Models

M1: $y_{i,t} = y^*_{i,t},$

M2: $y_{i,t} = \mu_i + y^*_{i,t},$

M3: $y_{i,t} = \mu_i + \beta_i t + y^*_{i,t},$

Initialization

$$y^*_{i,0} \sim \begin{cases} N(0, \frac{\sigma_i^2}{1-\rho^2}) & \rho \in (-1, 1) \\ O_p(1) & \rho = 1 \end{cases}.$$

Dynamic Estimation Bias

Background & New Issues

- **Common autoregressive bias source & exacerbation with intercept and trend**

  Orcutt (1949), Orcutt and Winokur (1969), Andrews (1993)

- **Panel autoregressive bias accentuated by pooling & effect of CS dependence**

  Phillips & Sul (2003)

- **Panel autoregressive estimates inconsistent in presence of individual effects & incidental trends**

  Nickell (1982), Neyman & Scott (1948), Moon & Phillips (1999)

- **Problems of Weak Instruments in IV & GMM estimation**

  Hahn & Kuersteiner (2000), Moon & Phillips (2004)

Weak Instrument Examples

- **Applied Microeconometrics:**
  earnings & schooling regressions

  Angrist & Krueger (1991, 2001)

- **Panel Models with Near Unit Roots**

  Hahn & Kuersteiner (2000)

  Moon & Phillips (2001, 2004)

$$y_{it} = \alpha_i + \left(1 + \frac{c}{T}\right) y_{it-1} + u_{it}$$

$$\Delta y_{it} = \left(1 + \frac{c}{T}\right) \Delta y_{it-1} + \Delta u_{it}$$

Instrument y_{it-2} **is weak because**

$$\Delta y_{it-1} = \alpha_i + \frac{c}{T} y_{it-2} + u_{it}$$

How does this affect inference?

Analysis of Firm Size

Gibrat's Law (proportional effect)

$$Z_{it} - Z_{it-1} = Z_{it-1} e_{it}, \text{ i.e. } z_{it} = z_{it-1} + e_{it}$$

Popular Empirical Formulation

Sutton (1997), Hall & Mairesse (2000)

$$z_{it} = \mu t + y_{it}, \; y_{it} = \rho y_{it-1} + \varepsilon_{it}, \; \rho \sim 1$$

Panel Model with Near Unit Root

$$\Delta z_{it} = \delta_i + \gamma_i' g_{pt} + \frac{c}{T} z_{it-1} + \varepsilon_{it}$$

Moon & Phillips (2004)

Implications

$$\frac{\partial \Delta z_{it}}{\partial z_{it-1}} = \frac{c}{T} < 0 \text{ if } c < 0$$

Dynamic Estimation Bias

Models M1, M2, M3: pooled estimator

$$\hat{\rho} = \rho + \frac{\sum_{t=1}^{T} \sum_{i=1}^{N} \tilde{y}_{it-1} u_{it}}{\sum_{t=1}^{T} \sum_{i=1}^{N} \tilde{y}_{it-1}^{2}}$$

Asymptotic Bias M2 – Nickell (1981)

$$plim_{N\to\infty}(\hat{\rho}-\rho) = G(\rho,T) = -\frac{1+\rho}{T-1} + O(T^{-2})$$

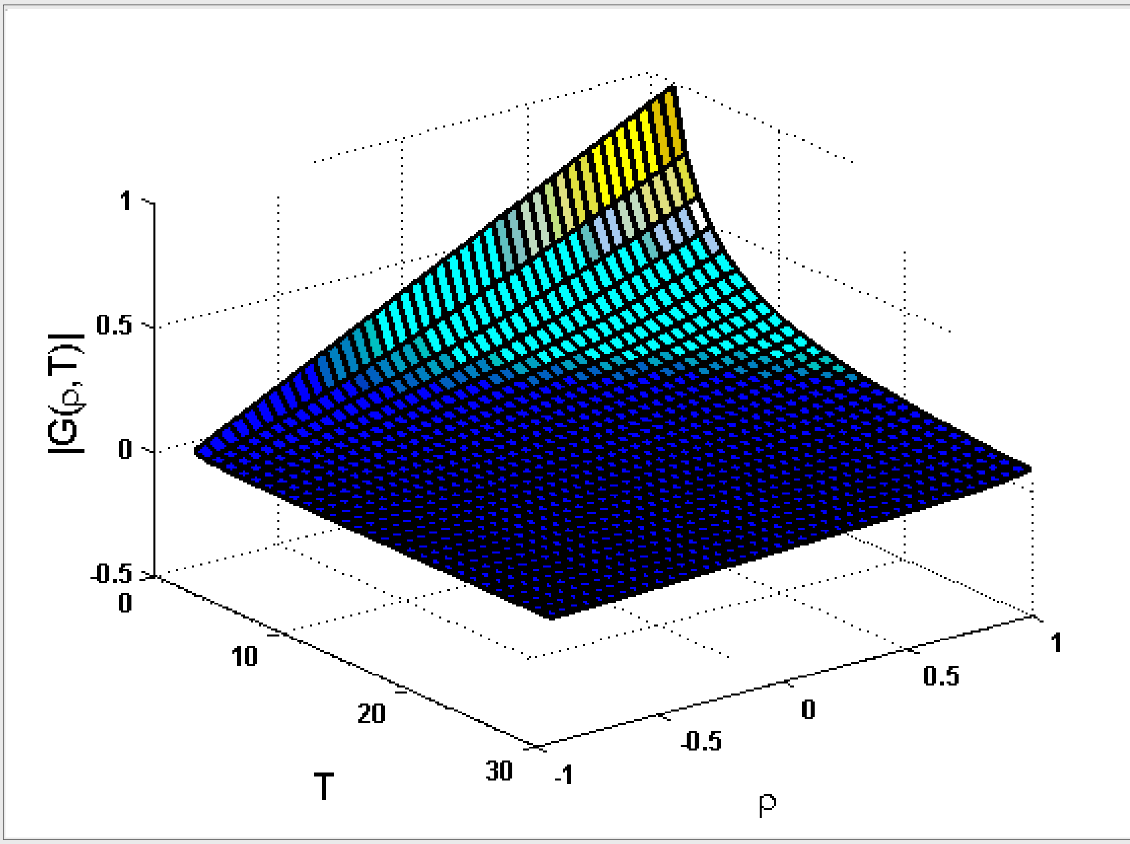
Unit Root Case M2

$$plim_{N\to\infty}(\hat{\rho}-1) = -\frac{3}{T+1}$$

also holds for heterogeneous case:

$$E(u_{it}^2) = \sigma_i^2, \quad \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 = \sigma^2$$

Inconsistency for Model M2



Asymptotic ($N \to \infty$) Bias Function $|G(\rho, T)| = -G(\rho, T)$ for Model M2.

Quantiles of Pooled OLS Estimator of $\rho = 0.9$

Sample	Model M1		Model M2		Model M3	
	5%	95%	5%	95%	5%	95%
N=1, T=50	0.710	0.962	0.628	0.937	0.548	0.904
N=1, T=100	0.787	0.948	0.749	0.935	0.713	0.920
N=10, T=50	0.858	0.928	0.799	0.889	0.735	0.843
N=10, T=100	0.874	0.920	0.847	0.902	0.820	0.882
N=20, T=50	0.872	0.921	0.816	0.880	0.755	0.831
N=20, T=100	0.882	0.915	0.857	0.896	0.830	0.874
N=30, T=50	0.878	0.917	0.824	0.875	0.763	0.825
N=30, T=100	0.885	0.913	0.861	0.893	0.835	0.870

$$\hat{\rho}_{pols} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1}-y_{i.-1})(y_{it}-y_{i.})}{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1}-y_{i.-1})^2}$$

For Model M2

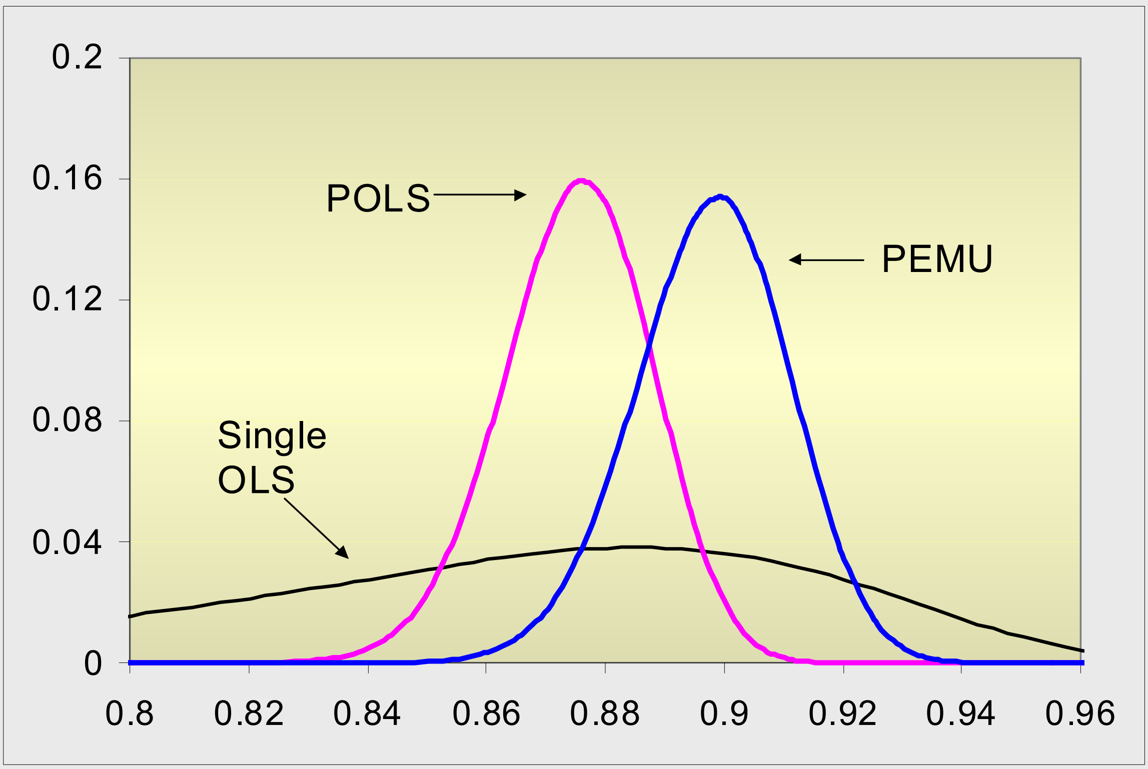
Implications for Estimation of Half-Life of Unit Shock

h = 6.5, ρ = 0.9

Sample	Model M1		Model M2		Model M3	
Quantile	5%	95%	5%	95%	5%	95%
N=1, T=50	2.027	18.036	1.487	10.730	1.153	6.905
N=1, T=100	2.890	13.034	2.403	10.393	2.051	8.342
N=10, T=50	4.532	9.244	3.086	5.897	2.248	4.071
N=10, T=100	5.130	8.332	4.184	6.753	3.487	5.518
N=30, T=50	5.313	8.019	3.573	5.171	2.561	3.614
N=30, T=100	5.698	7.617	4.645	6.095	3.847	4.973

$$\widehat{h} = \ln 0.5 / \ln \hat{\rho}_{pols}$$

Panel Autoregression density estimates



Empirical Distributions of Single Equation OLS, POLS and PEMU

No Cross Section Dependence

N = 20, T = 100, ρ = 0.9

Bias Reduction in Dynamic Panel Regression

- **Use Bias Correction Methods**

  - **asymptotic bias formulae –**
    Hahn & Kuersteiner (2002), Phillips & Sul (2003)

- **Median Unbiased Estimation**
  Lehmann (1959), Andrews (1993), Cermeno (1999), Phillips & Sul (2003)

  - **use invariance property & median function of panel pooled OLS estimator**
  - **median function**

$$m(\rho) = m_{T,N}(\rho)$$

  - **panel median unbiased estimator**

$$\hat{\rho}_{pemu} = \begin{cases} 1 & \text{if } \hat{\rho}_{pols} > m(1), \\ m^{-1}(\hat{\rho}_{pols}) & \text{if } m(-1) < \hat{\rho}_{pols} \leq m(1), \\ -1 & \text{if } \hat{\rho}_{pols} \leq m(-1), \end{cases}$$

Panel MU Estimation

- **Works well …. but**
  - **Uses Gaussianity**
  - **Need to have/find median functions by simulation**
  - **Is the median function increasing? Does the inverse function exist?**

$$m^{-1}(\hat{\rho}_{pols}),\ m^{-1}(\hat{\rho}_{pfgls})$$

  - **Is it Invariant?**
  - **What about more complex models?**

Model M3

Fitted Trend: pooled estimator bias

$$plim_{N\to\infty}(\hat{\rho}-\rho) = H(\rho,T) = -2\frac{1+\rho}{T-2} + O(T^{-2})$$

Unit Root Case M3

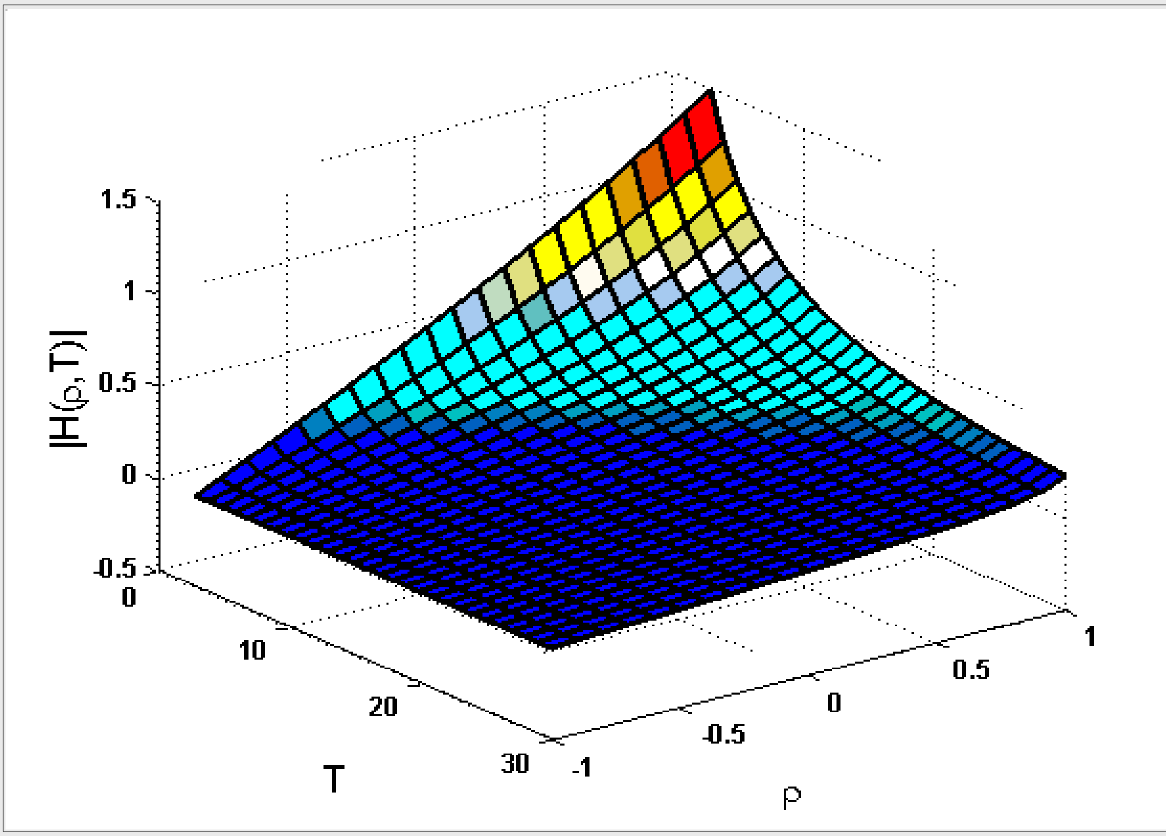
$$plim_{N\to\infty}(\hat{\rho}-1) = -\frac{7.5}{T+2}$$

holds in heterogeneous error case

inconsistency is > twice incidental trend case

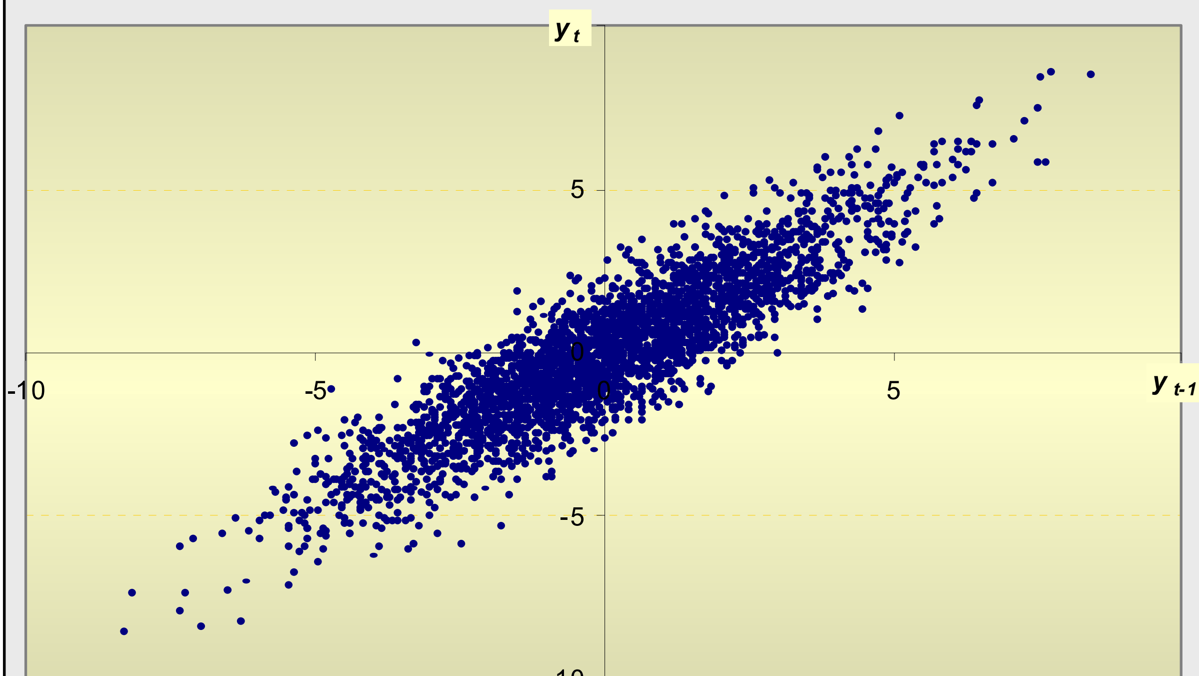
for T < 20, bias is very substantial

Inconsistency for Model M3



Asymptotic ($N \to \infty$) Bias Function $|H(\rho, T)| = -H(\rho, T)$ for Model M3.

Effect of Detrending Bias on Panel Data



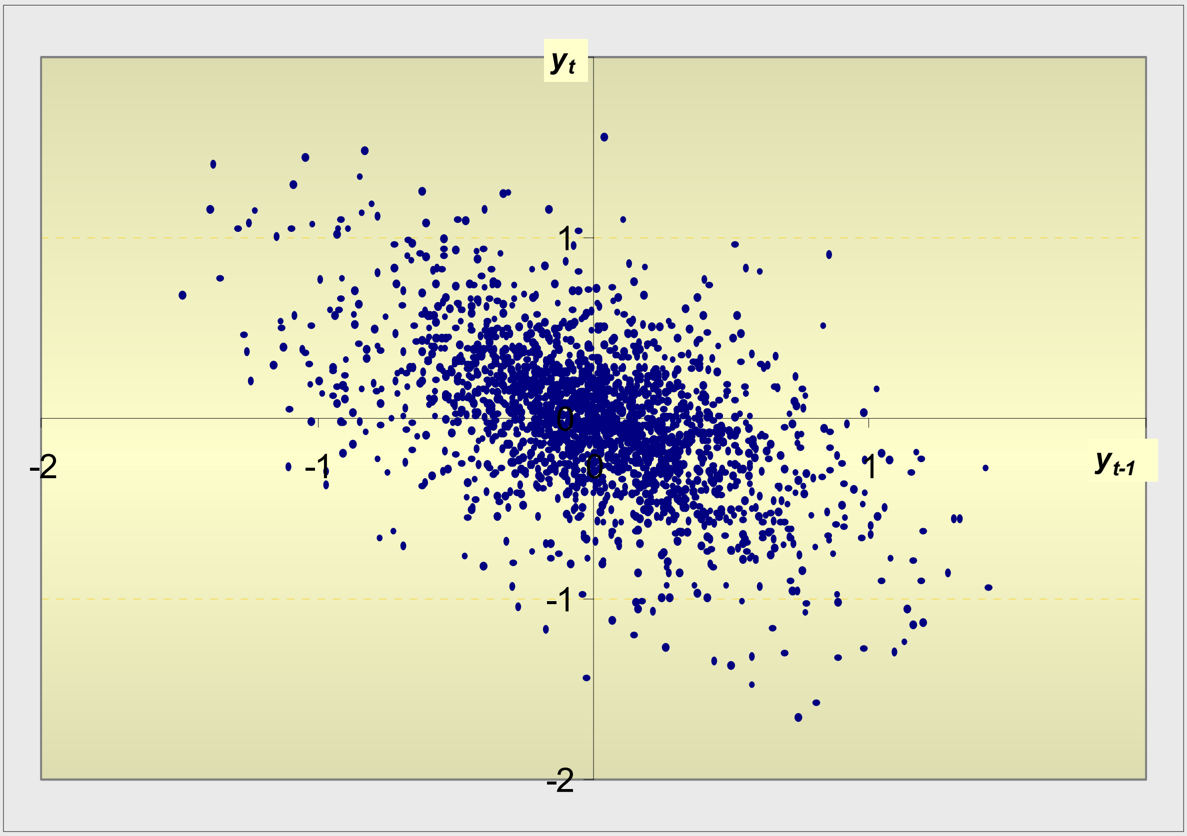
Sample Data before Detrending ($T = 4, N = 1,000, \rho = 0.9, \hat{\rho} = 0.90$)

Panel Model

$$y_{it} = \rho y_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim iid\ N(0,1)$$

$$t = 1, \ldots, T; \ i = 1, \ldots, N$$

After Detrending



Detrended Data ($T = 4$, $N = 1{,}000$; $\rho = 0.9$, $\bar{\rho} = \text{plim}_{N\to\infty}\hat{\rho} = -0.502$, $\hat{\rho} = -0.53$).

Panel Model

$$y_{it} = \rho y_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim iid\ N(0,1)$$

$$t = 1, \ldots, T; \; i = 1, \ldots, N$$

Models with Exogenous Variables

Model M4

$$\tilde{y} = \rho \tilde{y}_{-1} + \tilde{Z}\beta + \tilde{u}$$

Asymptotic Bias M4, $|\rho| < 1$

$$plim_{N\to\infty}(\hat{\rho} - \rho) = -\frac{\sigma^2 A(\rho,T)}{\sigma^2 B(\rho,T) + \beta'\left[plim_{N\to\infty}\frac{1}{N}\tilde{Z}'_{\rho,-1} Q_Z \tilde{Z}_{\rho,-1}\right]\beta}$$

$$\tilde{Z}^i_{\rho,t} = \sum_{j=0}^{\infty} \rho^j \tilde{Z}_{it-j}$$

$$plim_{N\to\infty}\left(\hat{\beta} - \beta\right) = -\left\{plim_{N\to\infty}\left(\tilde{Z}'\tilde{Z}\right)^{-1}\tilde{Z}'\tilde{Z}_{\rho,-1}\beta\right\} plim_{N\to\infty}(\hat{\rho} - \rho)$$

Models with Cross Section Dependence I

Model M2 + CSD

$$y_{it} = a_i + \rho y_{it-1} + u_{it},\ u_{it} = \sum_{s=1}^{K} \delta_{is}\theta_{st} + \varepsilon_{it}$$

where

$$\theta_{st}\ (s = 1, \ldots, K) = iid(0, \sigma^2_{s\theta})\ \text{over } t$$

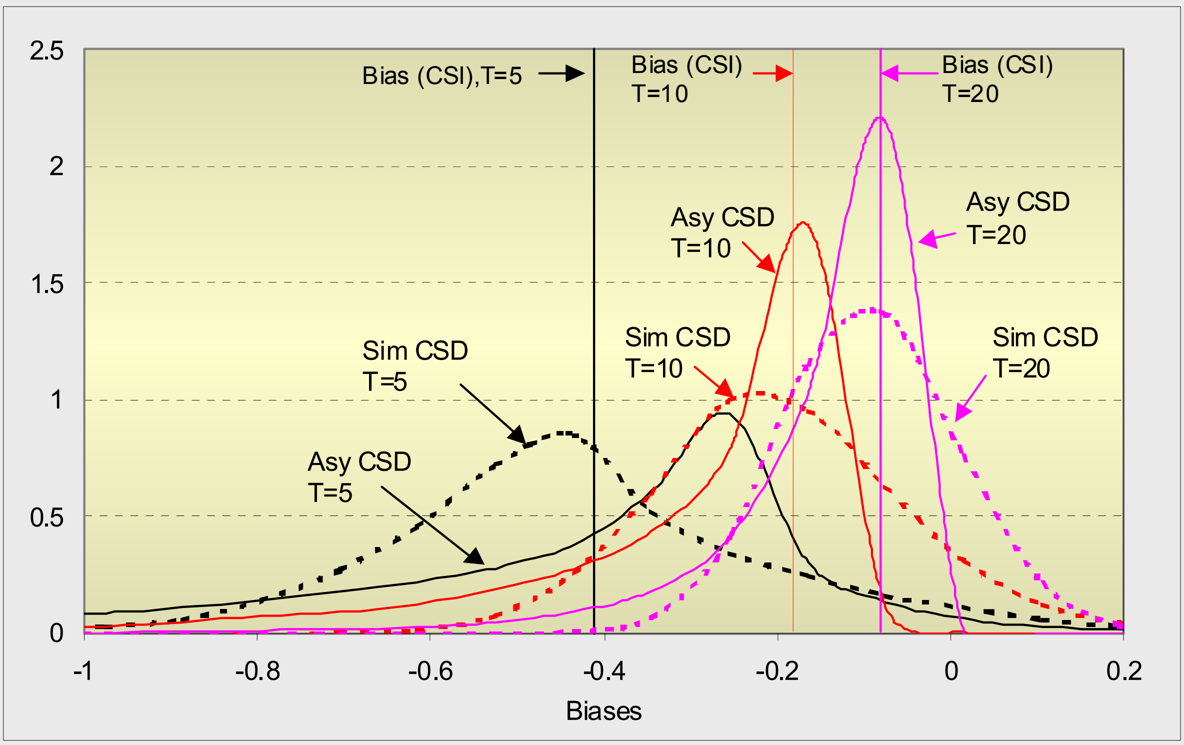
$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \delta^2_{si} = \mu^2_{\delta s}$$

Asymptotic Bias M2 + CSD, $|\rho| < 1$

$$plim_{N\to\infty}(\hat{\rho} - \rho) = -\frac{\sigma^2 A(\rho,T) + \psi_{AT}}{\sigma^2 B(\rho,T) + \psi_{BT}}$$

$$= -\frac{1+\rho}{T} - \frac{1+\rho}{T} \frac{\sum_{s=1}^{K} \mu^2_{\delta s}\sigma^2_{\theta s}\left(\eta^2_{\theta s} - 1\right)}{\sigma^2 + \sum_{s=1}^{K} \mu^2_{\delta s}\sigma^2_{\theta s}} + o_{a.s.}\left(\frac{1}{T}\right)$$

Random Inconsistency in Model M2 + CSD



Simulated (Sim) and Asymptotic (Asy) Distributions of Inconsistency of $\hat{\rho}$

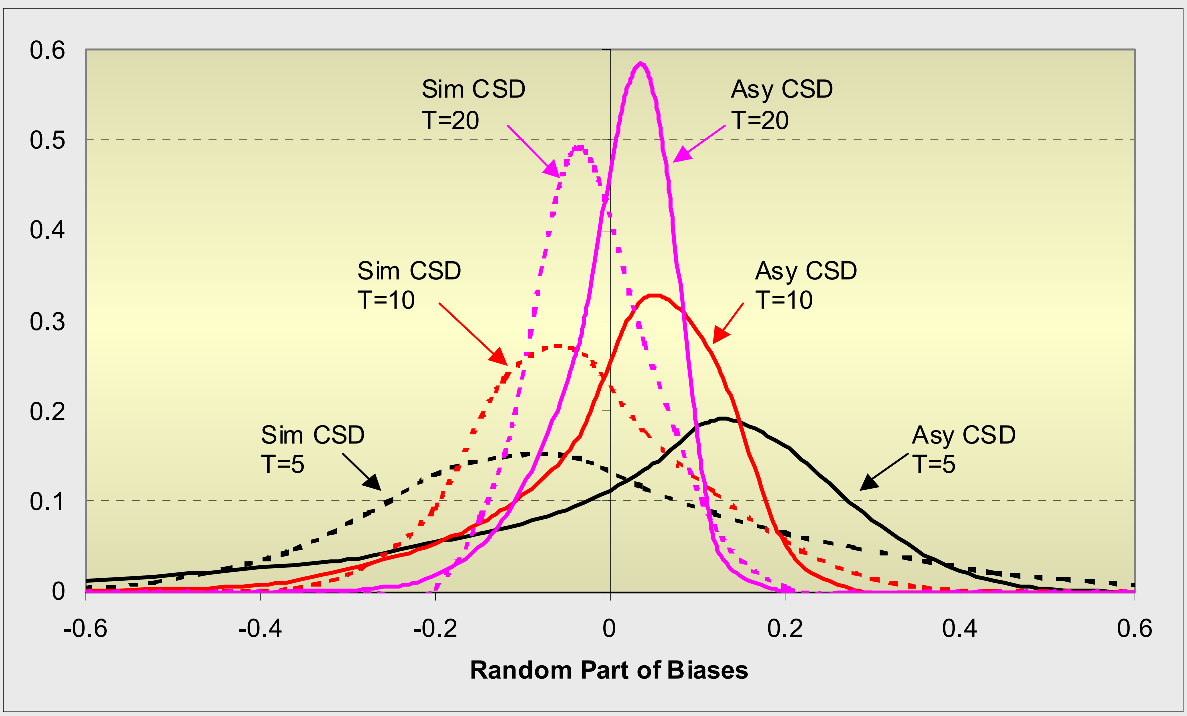
Simulations: N = 5,000, ρ = 0.5,

Unit Root Case

Asymptotic Bias M2 + CSD, $\rho = 1$

$$plim_{N\to\infty}(\hat{\rho} - 1) = -\frac{\sigma^2 A(T)+\phi_{AT}}{\sigma^2 B(T)+\phi_{BT}}$$

$$= -\frac{3}{T+1} - \frac{1}{T+1} g(W_s(r) : s = 1,\ldots,K) + o_{a.s.}\left(\frac{1}{T}\right)$$



Sim & Asy distributions of Random Parts of Inconsistency of $\hat{\rho}$

Models with Cross Section Dependence II

Model M3 + CSD

$$y_{it} = a_i + b_i t + \rho y_{it-1} + u_{it}$$

where

$$u_{it} = \sum_{s=1}^{K} \delta_{si}\theta_{st} + \varepsilon_{it}$$

Asymptotic Bias M3 + CSD, $|\rho| < 1$

$$plim_{N\to\infty}(\hat{\rho} - \rho) = -\frac{\sigma^2 C(\rho,T)+\psi_{CT}}{\sigma^2 D(\rho,T)+\psi_{DT}}$$

$$= -2\frac{1+\rho}{T} - \frac{1+\rho}{T}\frac{\sum_{s=1}^{K} \mu_{s\delta}^2 \sigma_{s\theta}^2 \left(\eta_s^2 - 2\right)}{\sigma^2 + \sum_{s=1}^{K} \mu_{s\delta}^2 \sigma_{s\theta}^2} + o_{a.s.}\left(\frac{1}{T}\right)$$

Unit Root Case

Asymptotic Bias M3 + CSD, *ρ = 1*

$$plim_{N \to \infty}(\hat{\rho} - 1) = -\frac{\sigma^2 C(T) + \phi_{CT}}{\sigma^2 D(T) + \phi_{DT}}$$

$$= -\frac{7.5}{T+2} - \frac{1}{T+2} h(W_s(r) : s = 1, \ldots, K) + o_{a.s.}\left(\frac{1}{T}\right)$$

Dealing with Bias & CSD Problems Together

- **Use GLS version of Panel MUE**
  - suitable for cases where feasible GLS possible
  - otherwise need to restrict dependence
- **Apply Panel feasible generalized MUE**
  - Step 1:

    Obtain $\hat{\rho}_{pemu}$ and error variance estimate $\widehat{V}_{pemu}$
  - Step 2:

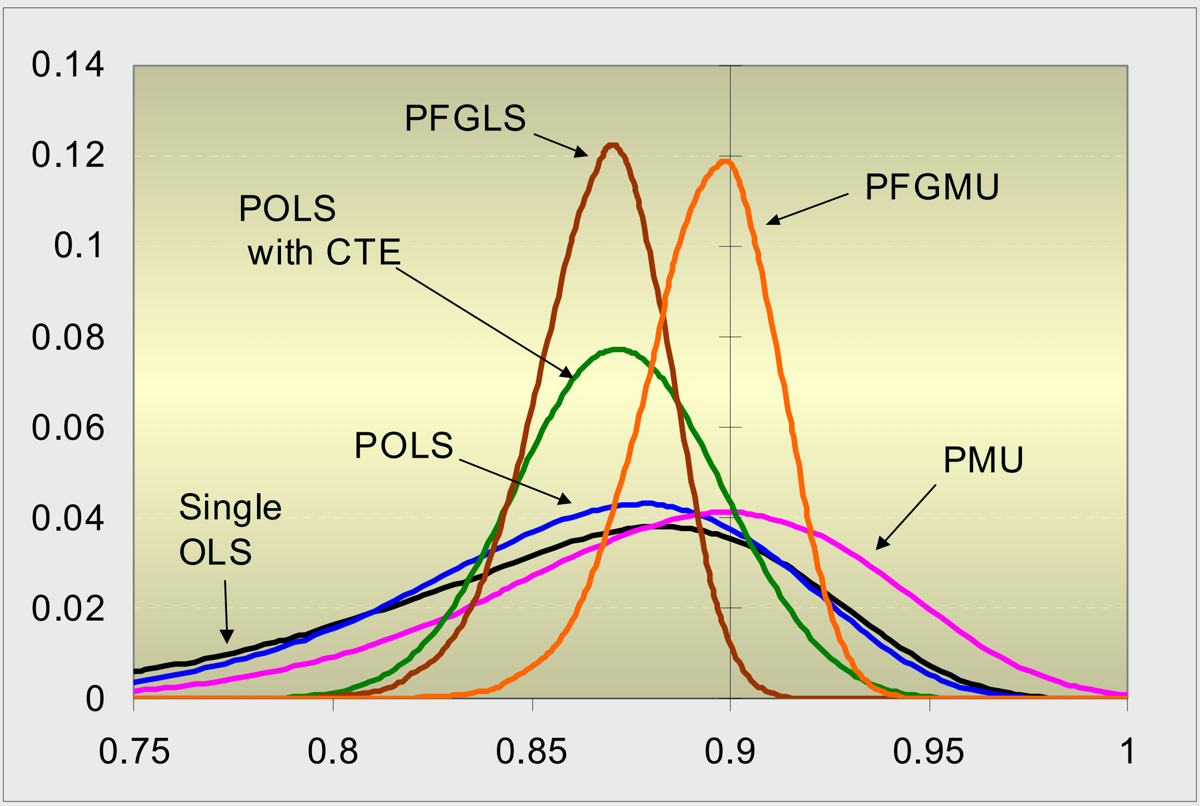
    Apply panel GLS

$$\hat{\rho}_{pfgls} = \frac{\sum_{t=1}^{T} \hat{y}'_{t-1} \widehat{V}_{pemu}^{-1} \hat{y}_t}{\sum_{t=1}^{T} \hat{y}'_{t-1} \widehat{V}_{pemu}^{-1} \hat{y}_{t-1}}$$

  - **Step 3:** Use its median function to calculate

$$\hat{\rho}_{pfgmu} = m(\hat{\rho}_{pfgls})^{-1}$$

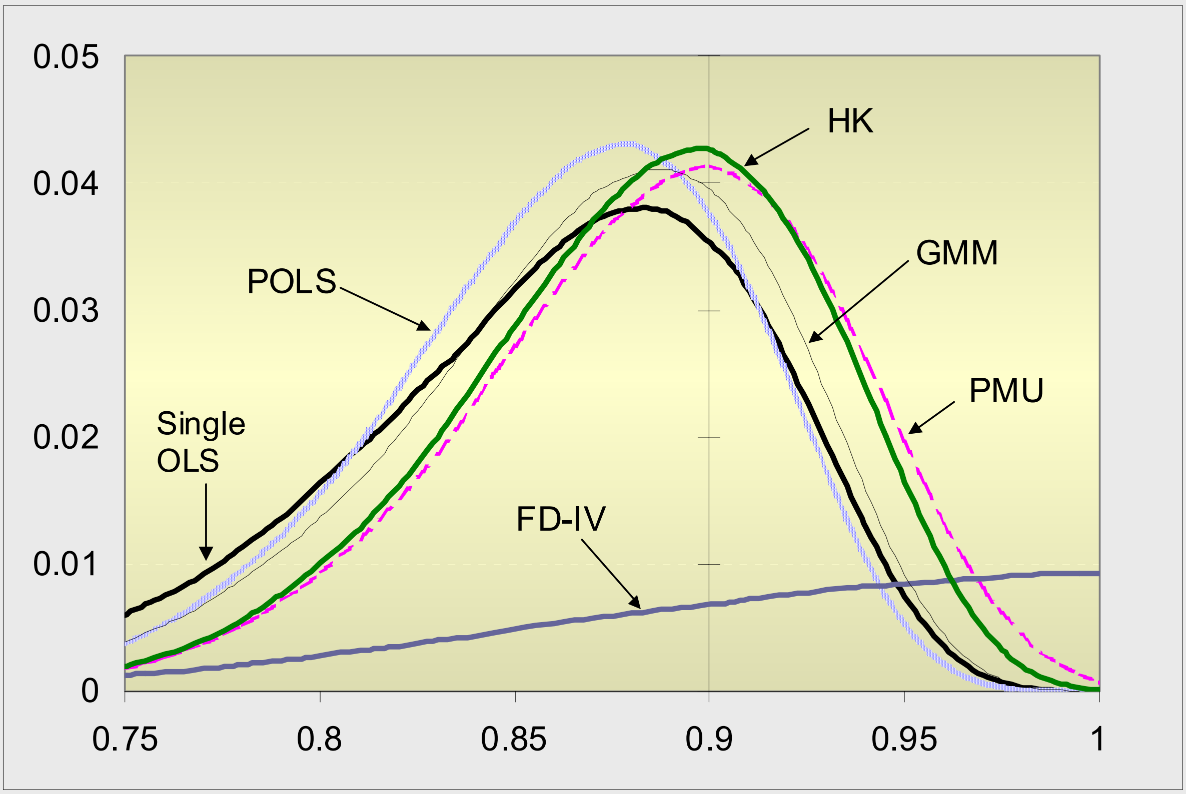
How Well Does PFGMU Work?



High Cross Section Dependence with $\delta_i \backsim iiU(1,4)$, $\rho(cross) \backsim 0.82$

N = 20, T = 100, ρ = 0.9

Comparison with other Bias Corrected Estimators



High Cross Section Dependence with $\delta_i \backsim iiU(1,4)$, $\rho(cross) \backsim 0.82$

N = 20, T = 100, ρ = 0.9

Panel MU Estimation under CSD

- Again, works well in simulations

  ….. but

  - Uses Gaussianity
  - Works when GLS feasible, so *N* must not be too large
  - Median function may not be invariant

$$\hat{\rho}_{pfgmu} = m^{-1}(\hat{\rho}_{pfgls})$$

  - Provides a benchmark

- Implications -

- Bias/inconsistency is important and can be huge for T small (< 10)

- Especially important when incidental trends are extracted

- Inconsistency is random when there is CSD. This raises dispersion.

- Need Bias correction + Variance reduction techniques

- Bias reduction relatively easy when no CSD:
  - plug in estimates into bias formulae, or
  - use inversion of bias function

  http://yoda.eco.auckland.ac.nz/~dsul013/mf.htm

- CSD case presents difficulties. Need to reduce dispersion by GLS methods (Phillips & Sul, 2003). But, as yet, no easy fix.

Empirical Application 1

- **Demand for Natural Gas** **Balestra–Nerlove, 1966**

$$G_{it} = \alpha_i + \underset{(0.063)}{0.68} G_{it-1} - \underset{(0.053)}{0.2} p_{it} - \underset{(0.022)}{0.014} \Delta M_{it} + \underset{(0.005)}{0.033} M_{it-1}$$

$$+ \underset{(0.008)}{0.013} \Delta Y_{it} + \underset{(0.01)}{0.004} Y_{it-1} + \text{error}$$

P = **relative price of gas,** **M** = **population,** **Y** = **income pc**

Autoregressive coefficient $\rho = 1 - r$, r = **depreciation**

Panel Regression Estimates: $\hat{\rho} = 0.68,\ \hat{r} = 0.32$

- **Bias corrections:**

  plug in method: $\hat{\rho} = 0.87,\ \hat{r} = 0.13$

  inversion method: $\hat{\rho} = 0.82,\ \hat{r} = 0.18$

Empirical Application 2

- **PPP deviations** **Frankel & Rose, 1996**

$$q_{it} = a_i + 0.88 q_{it-1} + \text{error}$$

q_{it} = log real exchange rate, T = 45, N = 150

- **Half life of PPP deviations**

$$h = \ln(0.5)/\ln(0.88) = 5.4 \text{ years}$$

- **Bias corrections:**

  plug in method: $\hat{\rho} = 0.92,\ h = 8.6$

  inversion method: $\hat{\rho} = 0.93,\ h = 10.2$

Time Series Unit Roots

- **Nonstationarity Tests**
  - Parametric tests (DF, ADF_t, ADF_a, SB)
  - Semiparametric tests (Z_t, Z_a, PS, VN)
  - Point optimal tests
  - QD/GLS (efficient) detrending procedures
  - Extensions to (non) cointegration testing
  - RRR model testing by LR
- **Stationarity Tests**
  - KPSS tests & parametric alternatives
  - Extensions to cointegrating testing
- **Model Selection Approaches**
  - Number of unit roots = order parameter
- **Fractional Alternatives**
  - Distinguishing short and long memory
  - Estimating memory semiparametrically
  - Testing nonstationarity: $d = 1$, $d \geq 1/2$

Overview of Panel Unit Roots

- **Nonstationarity Tests**
  - **Pooled P/NP tests (DF, ADF, VN-DW, PZ)**

    Quah, Levin-Lin, IPS, Phillips-Sul, Pedroni
  - **Allow for CSD & NP short memory**

    Phillips-Sul (2003), Moon & Perron (2003)
  - **Optimal/Point optimal tests**

    Ploberger-Phillips (2001), Moon, Perron, Phillips (2003)
  - **p-value tests (Maddala-Wu, Choi, Phillips-Sul)**
- **Stationarity Tests**
  - **Panel KPSS/LM test Hadri (2000)**
  - **Panel cointegrating testing McKoskey & Kao (1999)**
- **Model Selection Approaches**
  - **dynamic factors Bai & Ng (2002))**
  - **# unit roots = order parameter**
- **Fractional Alternatives**
  - **Some systems work, no panel analysis**

Panel Unit Root Tests under CSD

Testing Homogeneous Unit Roots

Under Unit Root Null with CSD

$$\frac{1}{\sqrt{T}} y_{[Tr]} = \frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} u_t \to_d B(r) \equiv BM(V_u)$$

$$B(r) = \delta B_\theta(r) + B_\varepsilon(r)$$

Apply Orthogonalization

$$\left[(\delta_\perp' \Sigma \delta_\perp)^{-1/2} \delta_\perp' \right] \frac{1}{\sqrt{T}} y_{[Tr]} \to_d (\delta_\perp' \Sigma \delta_\perp)^{-1/2} \delta_\perp' B(r)$$

$$(\delta_\perp' \Sigma \delta_\perp)^{-1/2} \delta_\perp' B_\varepsilon(r) = W_\perp(r) \equiv BM(I_{N-1}),$$

Modified Hausman Statistic

$$G_H^+ = T^2 \left(\underline{\hat{\rho}}_{emu}^{\,+} - \tilde{\rho}^+ i_{N-1} \right)' \left(\underline{\hat{\rho}}_{emu}^{\,+} - \tilde{\rho}^+ i_{N-1} \right)$$

where

$\underline{\hat{\rho}}_{emu}^{\,+}$	=	median unbiased estimates of ρ_i
$\tilde{\rho}^+$	=	PFGMU estimate of ρ

Moment-Based Estimation of δ, Σ

Orthogonalization

Numerical Optimization

$$\left(\hat{\delta},\hat{\Sigma}\right) = \arg\min_{\delta,\Sigma} tr[(M_T - \Sigma - \delta\delta')(M_T - \Sigma - \delta\delta')']$$

$M_T = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$, **from OLS or EMU residuals**

Iteration solving first order conditions

$\delta^{(r)} = (M_T\delta^{(r-1)} - \Sigma\delta^{(r-1)})/\delta^{(r-1)\prime}\delta^{(r-1)}$,
$\sigma_i^{(r)2} = M_{Tii} - \delta_i^{(r)2}$,

Orthogonalization Procedure

Construct $\hat{\delta}_\perp$ **and** $\widehat{F}_\delta = \left(\hat{\delta}_\perp'\hat{\Sigma}\hat{\delta}_\perp\right)^{-1/2}\hat{\delta}_\perp'$

$$\widehat{F}_\delta \to_p (\delta_\perp'\Sigma\delta_\perp)^{-1/2}\delta_\perp'$$

removes cross section dependence

Other Panel Unit Root Tests

based on orthogonalization

1. Cross section average statistics: G - tests

$$G_{ols}^{++} = \frac{1}{\sqrt{N}\sigma_\xi} \sum_{i=1}^{N-1} \left[\frac{\hat{\rho}_i^+ - 1}{\hat{\sigma}_{\hat{\rho}^+}} - \mu_\xi \right]$$

$$G_{emu}^{++} = \frac{1}{\sqrt{N}\sigma_{\xi^-}} \sum_{i=1}^{N-1} \left[\frac{\hat{\rho}_{i,emu}^+ - 1}{\hat{\sigma}_{\hat{\rho}_{i,emu}^+}} - \mu_{\xi^-} \right]$$

$$\rightarrow_d N(0,1)$$

$$\xi_i = \left(\int_0^1 W_i^2\right)^{-1} \left(\int_0^1 W_i dW_i\right),$$

$$E(\xi_i) = \mu_\xi, \ \mathrm{Var}(\xi_i)) = \sigma_\xi^2$$

c.f. Im, Pesaran & Shin (1997) used simulation to correct for bias

2. Tests based on p-values - Choi (2001)

$$P = -2\sum_{i=1}^{N-1} \ln(p_i), \quad Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N-1} \Phi^{-1}(p_i)$$

$$P \rightarrow_d \chi^2_{2(N-1)}, \quad Z \rightarrow_d N(0,1) \quad \text{as } T \rightarrow \infty, \text{ fixed } N$$

Simulation Performance of Panel Unit Root Tests

(correlation: min=0.52, med=0.82, max=0.94)

Model M2 - Fitted Intercept Case					
	Size: 5%				
Sample	IPS	G_{ols}^{++}	G_{emu}^{++}	P	Z
N=10,T= 50	0.257	0.052	0.052	0.044	0.046
N=30,T= 50	0.367	0.061	0.041	0.044	0.049
N=10,T=100	0.263	0.047	0.063	0.045	0.047
N=30,T=100	0.376	0.054	0.057	0.039	0.048

	Size Adjusted Power $\rho_i \sim U(0.8, 1.0)$				
Sample	IPS	G_{ols}^{++}	G_{emu}^{++}	P	Z
N=10,T= 50	0.247	0.252	0.270	0.997	0.996
N=30,T= 50	0.256	0.519	0.532	0.978	0.969
N=10,T=100	0.646	0.687	0.739	1.000	1.000
N=30,T=100	0.587	0.811	0.866	0.991	0.987

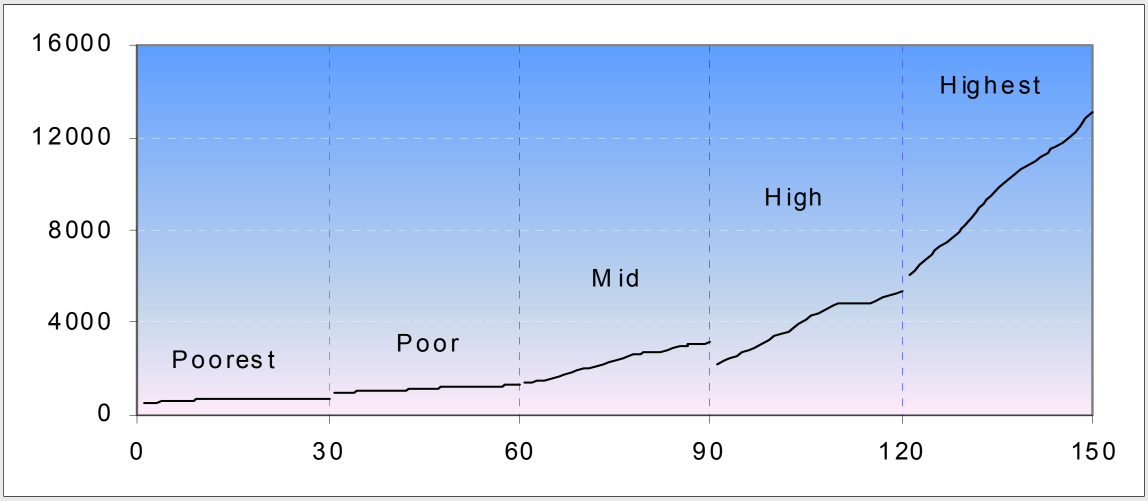
Simulation Performance of Panel Unit Root Tests

(correlation: min=0.52, med=0.82, max=0.94)

Model M3 - Fitted Intercept and Trend					
	Size: 5%				
Sample	IPS	G_{ols}^{++}	G_{emu}^{++}	P	Z
N=10,T= 50	0.278	0.077	0.072	0.043	0.048
N=30,T= 50	0.390	0.098	0.067	0.046	0.052
N=10,T=100	0.280	0.062	0.073	0.049	0.052
N=30,T=100	0.379	0.078	0.068	0.049	0.053

	Size Adjusted Power $\rho_i \sim U(0.8, 1.0)$				
Sample	IPS	G_{ols}^{++}	G_{emu}^{++}	P	Z
N=10,T= 50	0.122	0.086	0.088	0.985	0.983
N=30,T= 50	0.133	0.158	0.160	0.960	0.943
N=10,T=100	0.349	0.342	0.380	0.998	0.996
N=30,T=100	0.344	0.558	0.609	0.981	0.971

Economic Growth:
30 Years or 1,000 Years ?



Average Real per Capita Income over 1960-1989 with Country Groupings

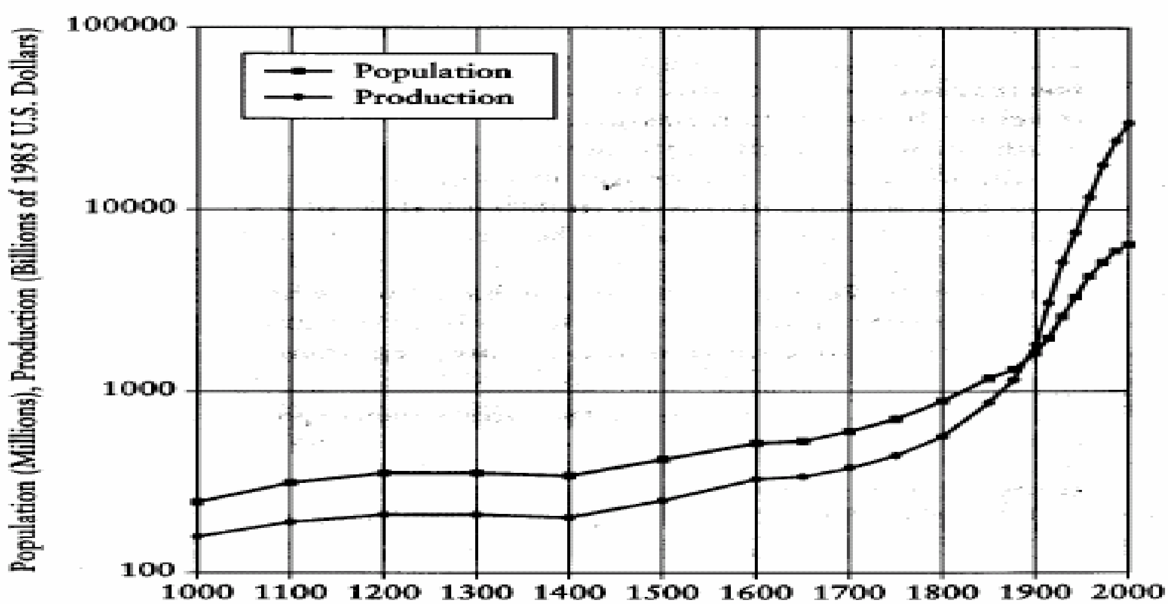


Figure 5.1 World population and production

Growth Convergence

Neoclassical Transition Dynamics

$$\log y_i(t) = \log\tilde{y}_i^* + [\log\tilde{y}_i(0)/\tilde{y}_i^*]e^{-\beta_i t} + \log A_i(0) + x_i t$$

Growth Convergence

Bernard & Durlauf (1995), Durlauf & Quah (1999)

$$\lim_{k\to\infty}(\log y_i(t+k) - \log y_j(t+k)) = 0$$

Requires

$$\lim_{t\to\infty} x_i(t) \to x, \quad \beta_i > 0$$

Issues

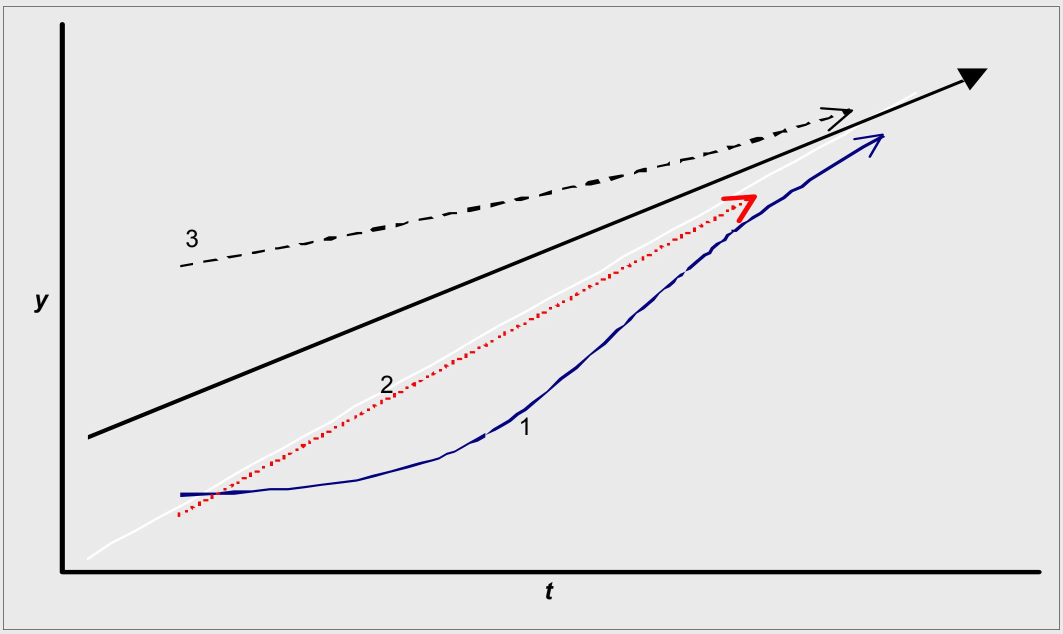
- **heterogeneity** $\beta_i \neq \beta_j$
- **initial technology conditions**

$$A_i(0) \neq A_j(0), \text{ or } A_i(0) = A(0)$$

- **time dependence**

$$x_i = x_i(t), \; \beta_i = \beta_i(t)$$

One Possible Scenario



Transitional Divergence and Ultimate Convergence

Panel Unit Root Analysis

Empirical Specification **Evans (1998), Bernard & Durlauf (1995)**

$$\log w_{it} - \log w_{\bullet t} = \alpha_i + \rho_i(\log w_{it-1} - \log w_{\bullet t-1})$$

$$+ \sum_{s=1}^{p_i} \omega_{is} \Delta\{\log w_{it-s} - \log w_{\bullet t-s}\} + u_{it}$$

$$\log w_{it} = \log y_{it} + v_{it}$$

Null

$$H_0 : \rho_i = 1 \text{ for ALL } i$$

Rejection does not imply overall convergence

Allow for CSD – one factor

$$u_{it} = \delta_i \theta_t + e_{it}$$

Empirical Results

Regional Convergence across US States 1929 - 1998

	G^+	Z	% of $\hat{\rho}_{emu} = 1$
	P-values		
All (48)	0.032	0.003	40
Subgroupings According to Income Level			
High (10)	0.282	0.259	33
Mid (17)	0.003	0.003	20
Low (21)	0.090	0.055	34
Subgroupings According to Cross-Sectional Error Correlation			
High (25)	0.361	0.071	100*
Mid (11)	0.005	0.019	27
Low (12)	0.262	0.136	43
Subgroupings According to Broad Regional Specification			
Northeast (16)	0.024	0.019	18
West (18)	0.000	0.004	17
South (14)	0.000	0.001	13

Econometric Modeling of Convergence

Model

$$\log y_{it} = b_{it}\mu_t + \varepsilon_{it}, \quad \varepsilon_{it} = a_i + \rho_i \varepsilon_{it-1} + u_{it}$$

Convergence Requires

$$C1 : \lim_{t\to\infty} b_{it} = b \text{ for } all\ i$$

$$C2 : |\rho_i| < 1 \text{ for } all\ i.$$

In transition

$$b_{it}\mu_t = b\mu_t + (b_{it} - b)\mu_t = b\mu_t + o(1), \quad \text{as } t \to \infty$$

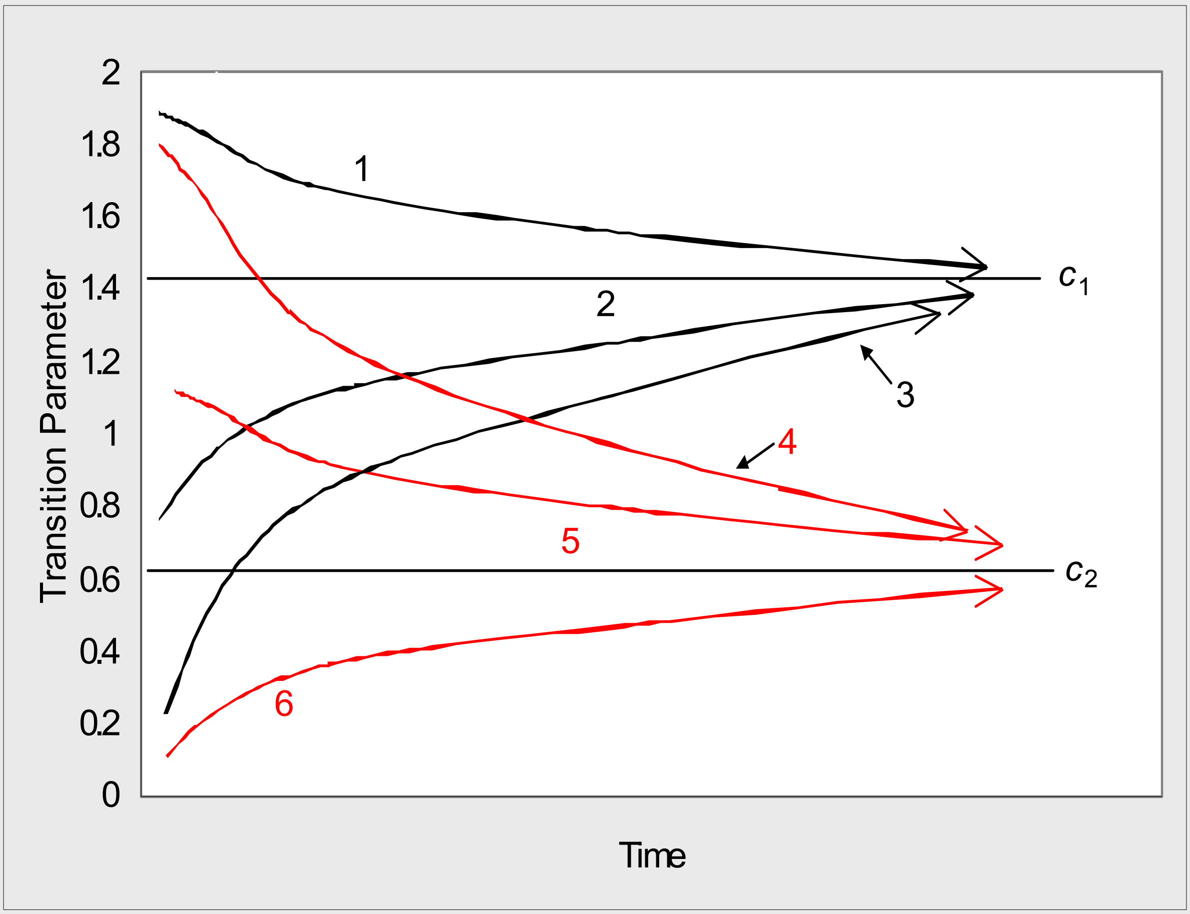
Transition parameter

$$h_{itN} = \frac{\log y_{it}}{\frac{1}{N}\sum_{i=1}^{N} \log y_{it}} = \frac{b_{it}}{\frac{1}{N}\sum_{i=1}^{N} b_{it}}$$

Test

$$\lim_{t\to\infty} h_{itN} = 1$$

Another Scenario



Conditional β-Convergence

Fitting the Transition Parameter

Use Whittaker HP filter

$$\hat{f}_{it} = \widehat{b_{it}\mu_t}$$

Take Cross Sectional Averages

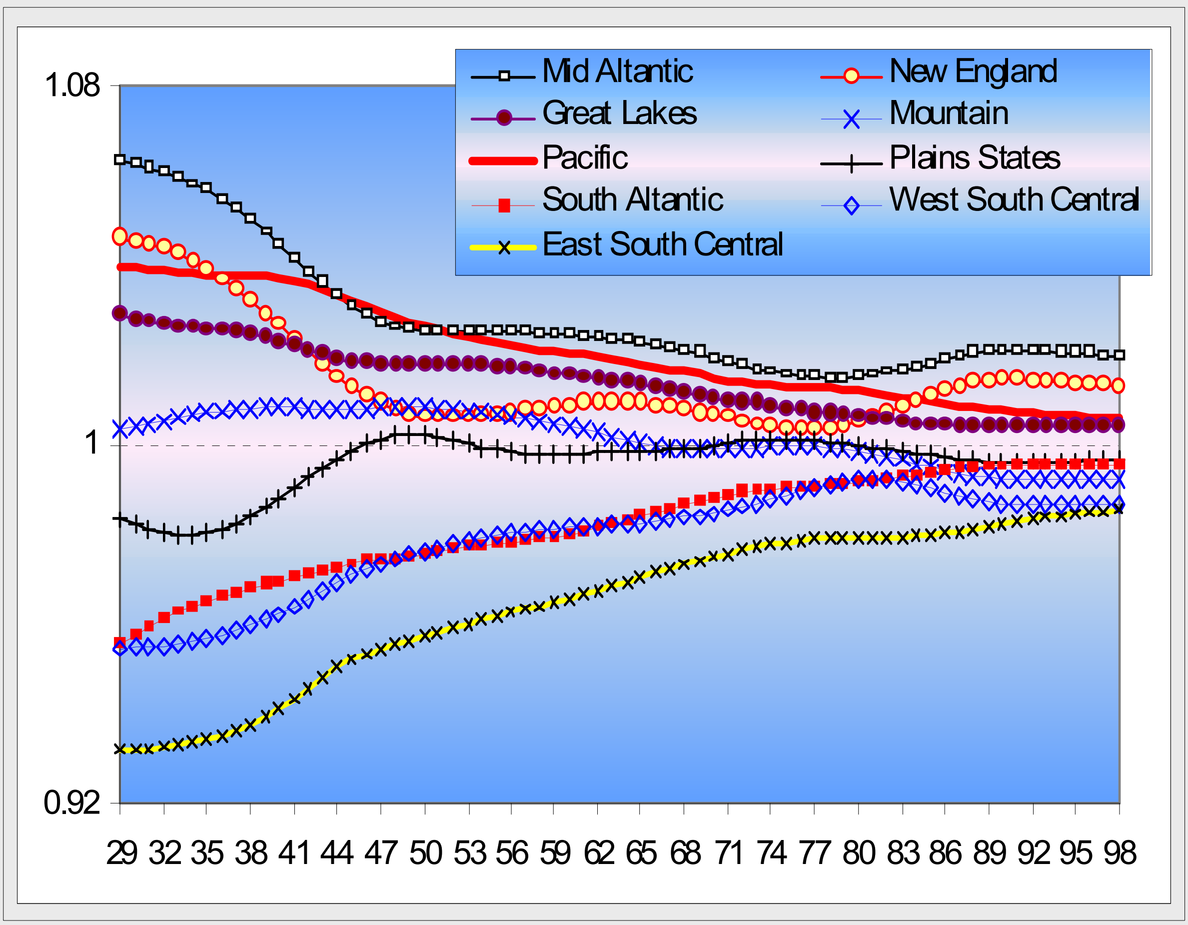
$$h_{it} = \frac{\hat{f}_{it}}{\frac{1}{N}\sum_{i=1}^{N}\hat{f}_{it}}$$

Error Analysis

$$\hat{f}_{it} = f_{it} + e_{it} = \left[b_{it} + \frac{e_{it}}{\mu_t}\right]\mu_t \qquad \frac{e_{it}}{\mu_t} = o_p(1)$$

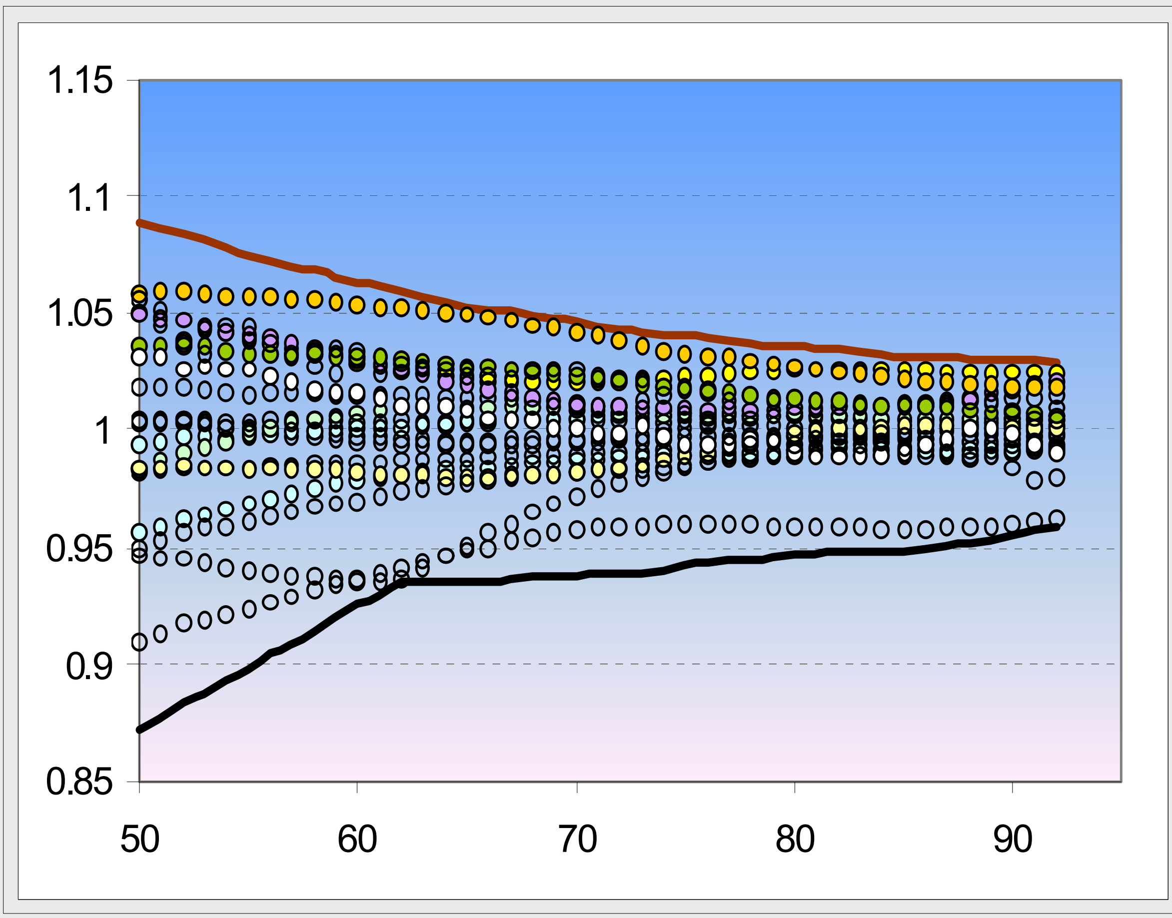
$$h_{it} = \frac{\left[b_{it}+\frac{e_{it}}{\mu_t}\right]}{\frac{1}{N}\sum_{i=1}^{N}\left[b_{it}+\frac{e_{it}}{\mu_t}\right]} \to_p 1, \quad \text{as } t \to \infty$$

Empirical Paths of Transition Parameters 1



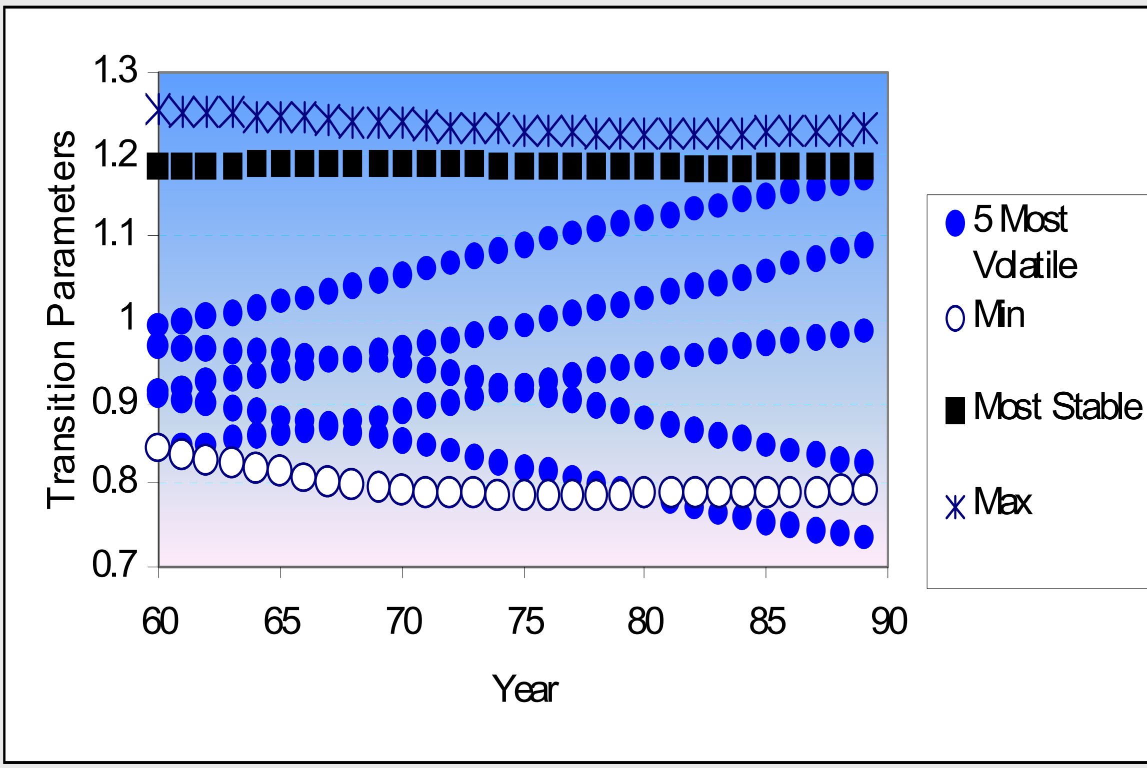
Time Profile of Regional Averages of Transition Parameters: 48 States.

Empirical Paths of Transition Parameters 2



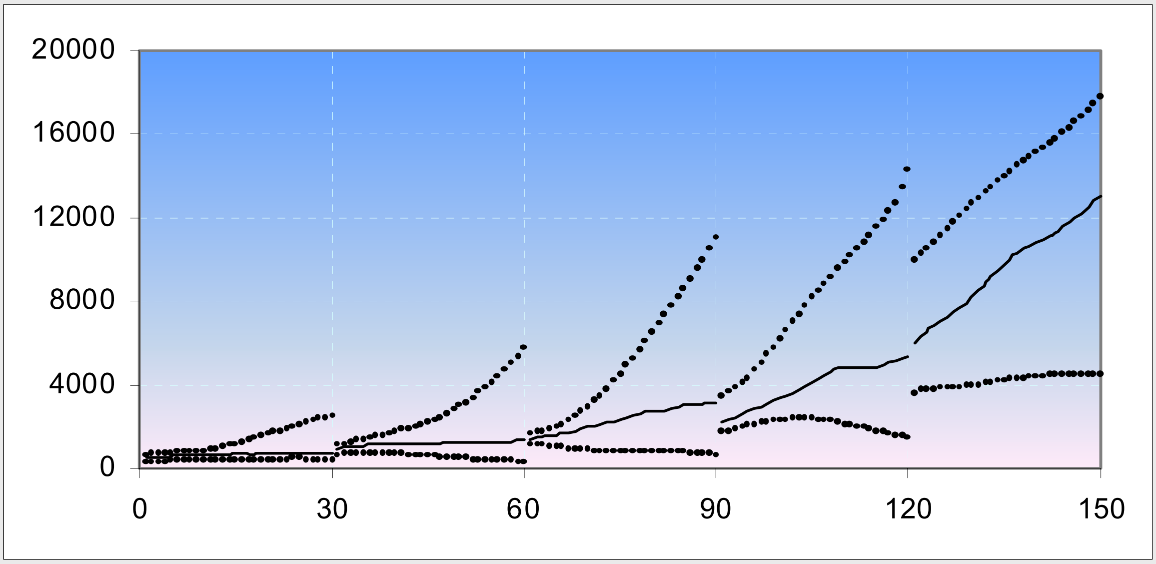
Transition Parameter Estimates: 21 OECD Countries 1950-1992.

Empirical Paths of Transition Parameters 3

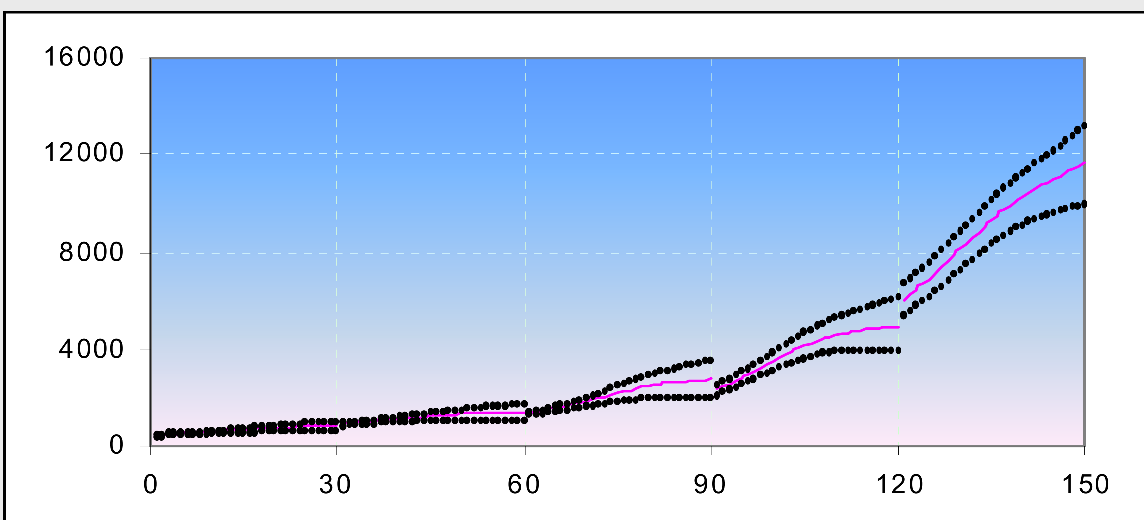


Transition Parameters for PWT (120 Countries 1960-1989)

Trajectories of p.c. Income within the Distribution



Mean, Min and Max trajectories of Distribution of Real pc Income 1960-1989



2.5%, 50% and 97.5% Quantiles (bootstrap) of Real p.c. Income 1960-1989.

- Conclude -

- **Dynamic panel bias can be substantial, especially when there are incidental trends**

- **CSD increases variance – even in the limit for large N. So bias reduction and variance reduction go hand in hand.**

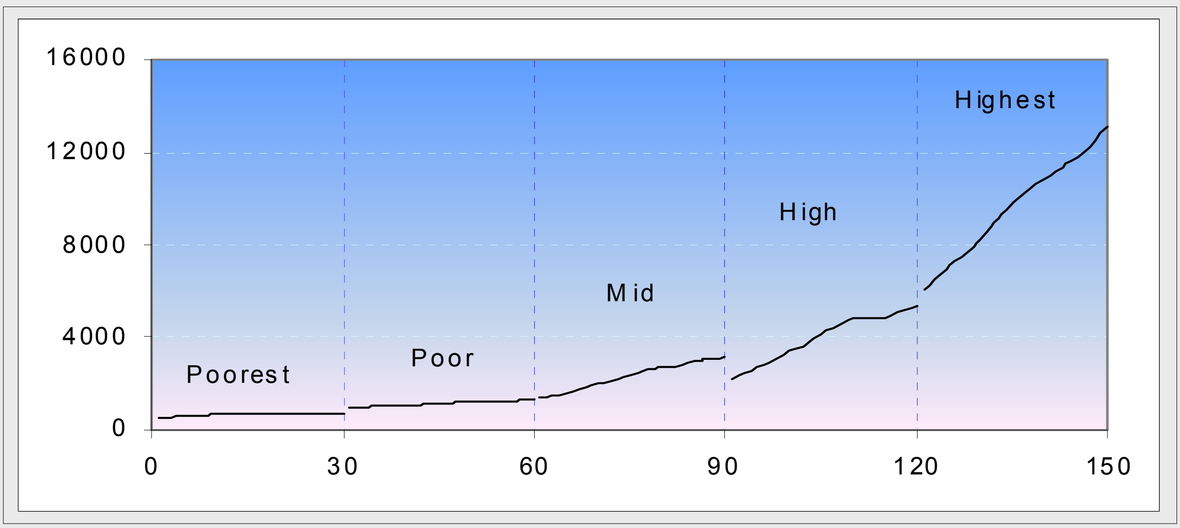
- **CSD affects panel unit root tests. This can be removed by suitable orthogonalization procedures.**

- **Point optimal panel unit root tests indicate that power is non trivial in $O(N^{-1/4})$ neigborhoods**

- **Need a wider tool kit than unit root tests to evaluate convergence and study transitions.**

- **Cross section averaging can conceal a great deal of variation**

New Methods for Time Series and Panel Econometrics



Average Real per Capita Income over 1960-1989 with Country Groupings

Peter C. B. Phillips

Cowles Foundation, Yale University

IMF Seminar: September 29, 2003